GUIDED NOTES (MODEL NOTES)

Vocabulary and Notation:

- Derivative: $D_x\left(F(x)\right) = f(x)$
- Antiderivative: $\int \underbrace{f(x)}_{\text{integrand}} \overbrace{dx}^{\text{“with respect to } x\text{”}} = F(x) + C$

integral sign

constant of integration

always add + C since $D_x(C) = 0$

Properties of Integrals:

$$\int k \cdot f(x)\,dx = k \cdot \int f(x)\,dx$$

We can factor out a coefficient.

$$\int \left[f(x) \pm g(x)\right] dx = \int f(x)\,dx \pm \int g(x)\,dx$$

We can take the derivative of each term.

Power Rule:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$\int 1\,dx = x + C$$

Find the general antiderivative for each of the following. *Integrate the function with respect to x.*

1) $f(x) = 3x^2 + 4$

$$\int \left(3x^2 + 4\right) dx$$

$$= \frac{3x^3}{3} + \frac{4x^1}{1} + C$$

$$= \boxed{x^3 + 4x + C}$$

2) $g(x) = 2x^2 + \pi x$

$$\int \left(2x^2 + \pi x\right) dx$$

$$= \frac{2x^3}{3} + \frac{\pi x^2}{2} + C$$

$$= \boxed{\frac{2}{3}x^3 + \frac{\pi}{2}x^2 + C}$$

Basic Trig Functions:

$$\int \sin x\,dx = -\cos x + C \qquad \int \cos x\,dx = \sin x + C$$

Find the general antiderivative for each of the following.

3) $f(x)=\sqrt{x}-\sin x$

$$\int\left(\sqrt{x}-\sin x\right)dx=\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}-(-\cos x)+C=\boxed{\frac{2}{3}x^{\frac{3}{2}}+\cos x+C}$$

Anti-Chain Rule:

$$\int f\left(g(x)\right)\cdot g'(x)\,dx = F\left(g(x)\right)+C$$

Evaluate each indefinite integral.

4) $\int 2(2x+1)^4\,dx=\boxed{\frac{1}{5}(2x+1)^5+C}$

$$u=2x+1 \qquad \frac{du}{dx}=2$$

$$du=2dx$$

$$\int u^4du=\frac{1}{5}u^5+C$$

Let u be what is raised to a power.

5) $\int\left(x^3-x\right)\sqrt{x^4-2x^2}\,dx=\boxed{\frac{1}{6}\left(x^4-2x^2\right)^{\frac{3}{2}}+C}$

$$u=x^4-2x^2 \qquad du=\left(4x^3-4x\right)dx$$

$$=4\left(x^3-x\right)dx$$

$$\int u^{\frac{1}{2}}\cdot\frac{1}{4}du=\int\frac{1}{4}u^{\frac{1}{2}}du=\frac{2}{3}\cdot\frac{1}{4}u^{\frac{3}{2}}+C=\frac{1}{6}u^{\frac{3}{2}}+C$$