Don't "u" Forget About C

Integration by Substitution

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Grade Level	12th Grade	**Time Frame**	85–100 minutes
Subject	Mathematics	**Duration**	2–3 class periods
Course	AP Calculus		

Essential Question

How can we find antiderivatives using our knowledge of derivatives?

Summary

In this lesson, students will use their knowledge of derivatives and pattern-recognition skills to find antiderivatives. Students will begin with the power rule for integration, evaluate the indefinite integral of basic trigonometric expressions, then learn how to integrate with u-substitution. Students are introduced to the idea of integration and its notation then apply their knowledge of antidifferentiation in a digital escape-room style activity. This lesson is intended to be used after completing derivatives and before the introduction of area and definite integrals. However, it can be adapted to be used later in the course.

Snapshot

Engage

Students use a table of derivatives and antiderivatives and use pattern recognition to complete the table, finding their first antiderivative.

Explore

Students recall the chain rule for derivatives, then using pattern recognition, attempt to "undo" the chain rule.

Explain

Students formalize their notation and understanding of antiderivatives and indefinite integrals.

Extend

Students practice finding antiderivatives and learn a little calculus history within an escape-room style Desmos Classroom activity.

Evaluate

Students demonstrate their understanding by creating their own composition function, finding its derivative, then trading to find the antiderivative of their peer's problem.

Standards

AP Calculus AB and BC Course and Exam Description (AP Calculus AB & BC (2020))

FUN-6.C.1: an indefinite integral of the function f and can be expressed where F'(x)= f(x) cand C is any constant.
FUN-6.C.2: Differentiation rules provide the foundation for finding antiderivatives.
FUN-6.D: For integrands requiring substitution or rearrangements into equivalent forms: (a) Determine indefinite integrals. (b) Evaluate definite integrals
FUN-6.D.1: Substitution of variables is a technique for finding antiderivatives.

Attachments

- Backward Beginnings—Don't u Forget About C - Spanish.docx
- Backward Beginnings—Don't u Forget About C - Spanish.pdf
- Backward Beginnings—Don't u Forget About C.docx
- Backward Beginnings—Don't u Forget About C.pdf
- Breaking the Chain—Don't u Forget About C - Spanish.docx
- Breaking the Chain—Don't u Forget About C - Spanish.pdf
- Breaking the Chain—Don't u Forget About C.docx
- Breaking the Chain—Don't u Forget About C.pdf
- Great Calculus Mystery (Teacher Guide)—Don't u Forget About C.docx
- Great Calculus Mystery (Teacher Guide)—Don't u Forget About C.pdf
- Great Calculus Mystery—Don't u Forget About C - Spanish.docx
- Great Calculus Mystery—Don't u Forget About C - Spanish.pdf
- Great Calculus Mystery—Don't u Forget About C.docx
- Great Calculus Mystery—Don't u Forget About C.pdf
- Guided Notes (Model Notes)—Don't u Forget About C.docx
- Guided Notes (Model Notes)—Don't u Forget About C.pdf
- Guided Notes—Don't u Forget About C - Spanish.docx
- Guided Notes—Don't u Forget About C - Spanish.pdf
- Guided Notes—Don't u Forget About C.docx
- Guided Notes—Don't u Forget About C.pdf
- Lesson Slides—Don't u Forget About C.pptx

Materials

- Lesson Slides (attached)
- Backward Beginnings (attached; one per student; printed front only)
- Breaking the Chain (attached; one per student; printed front only)
- Guided Notes handout (attached; one per student; printed front/back)
- Guided Notes (Model Notes) document (attached; for teacher use)
- Great Calculus Mystery handout (attached; one per student; printed front only)
- Great Calculus Mystery (Teacher Guide) document (attached; for teacher use)
- Index cards (one per student)
- Pencils
- Paper
- Student devices with internet access

10 minutes

Engage

Introduce the lesson using the attached **Lesson Slides**. Share the lesson's essential question on **slide 3** and the learning objectives on **slide 4**. Review each of these with your class to the extent you feel necessary.

Have students find a partner or assign partners, then display **slide 5**. Give each student a copy of the attached **Backward Beginnings** handout. Let students know that today they will be learning about antiderivatives, but do not yet tell students what antiderivatives are, as they will be using their pattern-recognition skills to try to answer this question themselves. Direct students' attention to the *I Notice* section of their handout. Have pairs work together to complete the table and find the missing antiderivative, using the pattern(s) they notice from the other rows of the table. As students work, monitor progress. For students who seem stuck, ask guiding questions, like the ones below, rather than directly telling them how to find the missing information.

- What are you sure/unsure of?
- What do we know about derivatives?
- What have you tried so far?
- Instead of trying to figure out how to go from the first column to the second column, what if we thought about it backwards and tried to figure out how to go from the second column to the first?

As pairs complete their tables, move to **slide 6** and ask for volunteers to share what they wrote for their last antiderivative. Then introduce the I Think / We Think strategy. Direct students' attention to the *I Think* portion of their handout. Ask students to quietly think about how they could describe the reasoning they used to complete the table and independently record their thinking on their handout.

***I Think*: Sample Student Responses:**

- Column 1 is the derivative of column 2, so I thought about what I could take the derivative of to get *x* to the five-halves power.
- I started in column 2 and took the derivative of each and got what was in column 1. Since we usually bring down the exponent and subtract 1 from the exponent, I instead added 1 to the exponent, but I'm not sure how to get the coefficient.

After a couple of minutes, have each student share with their partner their thinking then show **slide 7**. Direct students' attention to the *We Think* portion of their handout. Have pairs write a general rule describing how they completed the table. Let students know that the idea of a general rule (or generalizing) is that the written rule needs to work for all of the rows of the table, not just the last row of the table. After a couple of minutes, ask for volunteers to share their general rules. Use student responses to see if there are misconceptions. If the misconceptions are of derivatives, pause the lesson to address those prior-knowledge misunderstandings. If the misconceptions are of antiderivatives, be sure to address those during the Explain portion of the lesson.

***We Think*: Sample Student Responses:**

- If I add 1 to the exponent of what's in column 1, I get the exponent in column 2. It looks like I divide the coefficient in column 1 by that new exponent in column 2.
- To find a derivative, I know we multiply by the exponent, then subtract one from the exponent. Antiderivative sounds like the opposite of derivative, so I thought about opposite (inverse) operations. So to find the antiderivative, it looks like we add one to the exponent then divide by that new exponent.

10 minutes

Explore

Display **slide 8** and have pairs of students form small groups of 4. Give each student a copy of the attached **Breaking the Chain** handout and have students determine who is "Student 1," "Student 2," etc. This numbering simply determines the order of the tasks, but does not change anyone's difficulty of work. Introduce the class to the [Pass the Problem] strategy.

Move to **slide 9** and direct students to use the given composition function, $f(g(h(x)))$ to write $f(\ )$ in the second column of their row: Student 1 writes in row 1, Student 2 writes in row 2, etc. Then students pass their paper to the next person: Student 1 passes to Student 2; ...; and Student 4 passes to Student 1. On the paper they just received, have students write $g(\ )$ in the third column of the row that has been started.

Show **slide 10** and have students pass their papers again, in the same manner. On the paper they just received, have students write $h(x)$ in the fourth column of the row that has been started. Then have them pass their papers again and complete the row by finding the derivative of the composition function (using the chain rule).

Display **slide 11** and direct students to pass their paper back to the original owner. Have students repeat these steps again for the next row, while skipping the fifth row: Student 1 starts row 2; ...; and Student 4 starts row 1.

Once students have their original papers back a second time, transition to **slide 12**. Have students complete the remaining two rows, still skipping the fifth row. Once students are finished, have students check their work within their group.

Teacher's Note: Guiding the Activity

In the third row, help students understand what is being squared: just $\pi \cdot x$. Consider having students rewrite the expression with an extra set of parentheses to emphasize that the sine function is not being squared but that πx is.

As students are checking their work, transition through **slides 13–14** so groups can check their work. Give students time to ask clarifying questions about the chain rule for derivatives.

Move to **slide 15** and tell students that their challenge is to use the given derivative in the last row to complete the table. Have students work quietly and independently for a few minutes before allowing them to discuss with their group.

Show **slide 16** and share the composition function that goes with the given derivative. Ask for volunteers to share how they figured out the original composition function. Then ask if they know what word they might use to describe the function in column 1 if the derivative is in the last column. Use this to transition to the Explain portion of the lesson where students learn more about *antiderivatives*.

20 minutes

Explain

Teacher's Note: Looking Ahead

Notice where the *u-du* scratchwork is on the Guided Notes (Model Notes) document. This is intentionally written as scratch work and is not written as equal to the given integral. This approach is also helpful with definite integrals. When working with definite integrals, this approach means that the bounds do not need to change when the variables change, because the *u-du* scratchwork uses an indefinite integral. Please keep in mind that when students learn about definite integrals with *u*-substitution, it is important that students do understand how to change the bounds, as that skill may be assessed on the multiple-choice portion of the AP exam.

If you are modifying this lesson to be taught after introducing definite integrals, please be sure to share with students how to change the bounds and the importance of making accurate statements—a definite integral likely does not equal an indefinite integral.

Show **slide 17** and give each student a copy of the attached **Guided Notes** handout. Use the slide to explain to students the meaning of the notation for antiderivatives/integrals. Be sure to emphasize the use of *antidifferentiation* being the same as *integration* and that *general antiderivatives* are the same as *indefinite integrals*.

Move to **slide 18**, then complete the Guided Notes handout with the class. Ask students to recall the activity from the Engage portion of the lesson and have them tell you how to write the power rule for integrals. Similarly, instead of telling students the antiderivative of the sine and cosine functions, ask them to tell you what they should be. Be sure for each example to emphasize the importance of the constant of integration (+C). Use the attached **Guided Notes (Model Notes)** document as needed.

Teacher's Note: Guiding the Lesson

As students are learning about *u*-substitution, a common question is “How do we know what to pick for *u*?” Share with students that knowing the answer to that will come with practice, so it is okay to “pick the wrong *u*,” try it out, and find out that they “picked too much” or “picked the wrong *u*.” A helpful guideline to share is to suggest that they think about the Explore activity: *u* is often what is inside of that “outside” function, *f*( ). In general, *u* is often within a trigonometric function or is an expression taken to a power.

For students who struggle to pick *u*, consider giving the advice of having them find the “scariest” part of the integrand, then let *u* be what is inside of that “scary part” of the integrand.

35 minutes

Extend

Teacher's Note: Desmos Classroom Activity Preparation

To use this Desmos Classroom activity, select the following link: "Don't "u" Forget About C." Create an account or sign in under the "Activity Sessions" heading. After you log in, the "Assign" dropdown button will be active. Click the arrow next to the word "Assign," then select "Single Session Code." After making some setting selections, select "Create Invitation Code" and give the session code to students. For more information about previewing and assigning a Desmos Classroom activity, go to https://k20center.ou.edu/externalapps/using-activities/.

For more detailed information about Desmos features and how-to tips, go to https://k20center.ou.edu/externalapps/desmos-home-page/.

Show **slide 19** and provide students with your session code. Then, have students go to student.desmos.com and enter the session code.

Teacher's Note: Sign-in Options

If students sign in with their Google or Desmos accounts, then their progress is saved, and they can resume the activity or view their work later. If students continue without signing in, they can complete the activity, but they must do so in one sitting. It is strongly recommended that students sign in; otherwise, they risk losing their work.

Give each student a copy of the attached **Great Calculus Mystery** handout and have them get a piece of notebook paper and pencil. Introduce the activity using **screen 1**, which shares how this escape-room style activity will function. Remind students that they will need to show their work on their notebook paper and be expected to turn in this scratch work when they are finished.

Have students work independently in pairs or individually through the activity. Here students are asked to find general antiderivatives of basic functions, identify *u* and *du*, rewrite integrals using *u*-substitution, and lastly evaluate indefinite integrals. Use the attached **Great Calculus Mystery (Teacher Guide)** document for more details and support of this activity.

Teacher's Note: Assessing Scratch Work

As students become more and more proficient at integration, students may show less and less work. However, remind them that for the AP exam it is important to practice showing work, which may be that they only write *u* and *du* and integrate in their head. However, some students may feel that on some problems they do not need to write that down because it is "obvious." Encourage students to at least write down what *u* and *du* are for at least this first lesson. From there, clearly communicate your expectations.

10 minutes

Evaluate

Use the Exit Ticket strategy to individually assess what students have learned from the lesson. Display **slide 20** and have each student create a composition function, writing it on the back of their Great Calculus Mystery handout. Then have students find the derivative of their own function. Encourage students to not create composition functions that will require the product or quotient rule to find the derivative, and if they did, tell them to pick a different composition function. As they work, give each student an index card. Have students write the simplified derivative on their index card, labeling it $h'(x)$.

Transition to **slide 21**. Have students trade index cards and evaluate the indefinite integral they have been given, labeling their antiderivative $h(x)$.

Once they are done, show **slide 22** and have students trade their index cards back for their classmate to check their work, comparing it to their original composition function. If $h(x)$ does not match the original composition function, have students work together, sharing their work and thinking for both the derivative and antiderivative to find the error.

Resources

- K20 Center. (n.d.). Bell ringers and exit tickets. Strategies. https://learn.k20center.ou.edu/strategy/125
- K20 Center. (n.d.). Desmos classroom. Tech Tools. https://learn.k20center.ou.edu/tech-tool/1081
- K20 Center. (n.d.). I think / we think. Strategies. https://learn.k20center.ou.edu/strategy/141
- K20 Center. (n.d.). Pass the problem. Strategies. https://learn.k20center.ou.edu/strategy/151