K20 LEARN

Gummy Bears in Space

Bivariate Data Analysis and Regression Models

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Grade Level	11th – 12th Grade	**Time Frame**	45 - 90 minutes
Subject	Mathematics	**Duration**	1 - 2 days
Course	AP Statistics		

Essential Question

How can we use bivariate data and regression models to analyze and predict the motion of launched objects?

Summary

In this statistics lesson, students learn how to set up a catapult, launch objects, and determine which object is most reliable. They develop experimental designs and perform various calculations, including a 5-number summary and a modified box-and-whisker plot, to analyze the data through regression models and to determine the most reliable object to launch through the lens of launching astronauts (gummy snacks and ping-pong balls) across the classroom.

Snapshot

Engage

Students will set up their catapults and practice using them by launching gummy snacks and ping-pong balls.

Explore

Students will work in groups to launch their “astronauts” (gummy snacks and ping-pong balls), measuring and recording the distance traveled.

Explain

Students create a 5-number summary and modified box and whiskers plot before reviewing experimental design factors and determining their most consistent astronaut.

Extend

Groups manipulate factors such as launch angle, pull-back angle, and paper plate distance to create an accurate model that can predict an astronaut’s traveled distance.

Evaluate

Groups use their models to predict the launch and pull-back angle needed to land their astronaut on a paper plate at a fixed distance.

Standards

AP Statistics Course and Exam Description (Course at a Glance)

1.5: Representing a Quantitative Variable with Graphs
1.6: Describing the Distribution of a Quantitative Variable
1.7: Summary Statistics for 2 a Quantitative Variable
1.8: Graphical Representations of Summary Statistics
1.9: Comparing Distributions of a Quantitative Variable
2.4: Representing the Relationship Between Two Quantitative Variables
2.6: Linear Regression Models
2.7: Residuals
2.8: Least Squares 2 Regression
2.9: Analyzing Departures from Linearity

Attachments

- Lesson Slides—Gummy Bears in Space.pptx
- Mission Log - Gummy Bears in Space.docx
- Mission Log - Gummy Bears in Space.pdf
- Xpult Data (1).xlsx

Materials

- Lesson Slides (Attached)
- Mission Log Handout (one per student)
- Whiteboard or projector
- Paper plates (one per group, used as a landing target)
- Masking tape or painter tape (for marking launch positions)
- Xpult Catapult (1 per group)
- Ping pong balls (multiple per group, used as one of the "astronauts")
- Gummy snacks (multiple per group, used as the second "astronaut")
- Metric tape measure (one per group)
- Graphing calculators (TI-84 or similar) *or* laptops with spreadsheet software (for data analysis)
- **Optional:** Xpult Data Spreadsheet (attached for regression modeling)
- **Optional:** Colored pencils or markers (for distinguishing between different data sets)

Preparation

Equipment

This lesson is written using the Xpult Science Project Catapult. For more information on setting up the catapult and making adjustments, please visit the company's FAQ.

Setting up the Classroom

Before the lesson starts, arrange the classroom so that each group has enough space to launch their objects using the catapults. This can be achieved by moving to a gym, cafeteria, or other open indoor space in the school or by creating "firing lanes" in the classroom. Each lane should be 3-5 feet wide and span the longest length of the classroom.

Setting Expectations

This activity involves launching objects across the room, students moving to measure distances, and active group discussions. Before beginning, establish clear expectations for the following:

- How many projectiles will each group receive? Approximately 5-10 gummy snacks and 2-4 ping pong balls per group.
- How will groups keep track of their objects? Consider numbering ping pong balls with group numbers and assigning a specific color gummy snack to each group.
- How will you get the class’s attention when necessary? Consider using a class call back or playing a bell or chime from the computer.
- How can students collaborate without disrupting other groups? Consider assigning groups a corner of the room for their group discussion.

15 minutes

Engage

Use the **Lesson Slides** to guide the lesson. Start by posing the essential question and explaining the objectives found on **slides 3–4**.

Transition to **slides 5 & 6** to introduce the catapult and to explain its mechanisms, functions, and setup. Divide the class into groups of 3–4 students and assign each group a catapult and classroom space.

Once groups are assigned and catapults are set up, place a paper plate about 2.5 meters (approximately 8 feet) away from each catapult as a target.

Provide each group with ping pong balls and gummy snacks. Set a 5-minute timer and instruct students to begin launching their “astronauts,” adjusting the pull-back angle to attempt to land their object on the target (paper plate). At this point in the lesson, students are exploring how changing conditions on the catapult will affect the distance that their astronauts travel.

30 minutes

Explore

Transition to **slide 7,** distribute the attached **Mission Log handout** to each student and provide each group with a metric tape measure. Display **slide 8** and instruct students to mark the location of their catapult on the floor using a piece of masking tape.

Display **slide 9** and show students how to adjust the launch angle to 45° and the pull-back angle to 60° so that all launches follow the same conditions.

Next, use **slide 10** to assign roles within each group:

- **Recorder**: This student will record all measurements and data collected.
- **Launcher**: This student will operate the catapult.
- **Measurers (1-2 students)**: These students will use the metric measuring tape to measure the horizontal distance traveled by the ping pong ball and gummy snacks. Another student will be responsible for retrieving the projectiles.

Inform the class that they will conduct 10 trial launches for both a ping pong ball and a gummy snack (the astronauts) and measure the landing distance from the catapult, which is the point where it first makes contact with the ground. Once students have been assigned their roles, display **slide 11** and play the five-minute timer. Have students use this time to practice launching 2-3 gummy snacks to ensure everyone understands their assigned task.

Once all groups feel comfortable with the process, instruct them to use the Explore portion (page 1) of their Mission Log handout to record their data carefully for analysis later in the lesson.

25 minutes

Explain

Once each group has recorded their data for 10 trials, transition to **slide 12.** Instruct students to complete the Explain part A on their handout (page 2) individually by calculating a 5-number summary. Students can then identify the interquartile range (IQR), determine the left and right fences, and create a modified box-and-whisker plot for each treatment.

After completing these calculations, have students discuss their findings in their groups and respond to prompts B-H on the Explain portion of their handout (page 2), which guides them in analyzing their comparative data with the following questions:

1. Write a few sentences comparing the shape, center, spread, and unusual features of the two plots. Be sure to use appropriate metrics for the center and spread.
2. Use an appropriate statistic(s) to describe which astronaut typically flies further.
3. Use an appropriate statistic(s) to describe which astronaut is the most consistent.
4. Which statistic would you consider the most important metric in evaluating the performance of your astronauts?
5. Which astronaut would you use?
6. What are the factors we can manipulate? At what levels?
7. What is our response variable?

Teacher's Note: Guiding the Lesson

Mean and standard deviation can only be used with unimodal and symmetrical data without outliers. For all other data, we must use median and IQR as the metric for center and spread.

Consider reviewing this concept with students before showing **slide 12** by asking them to identify appropriate metrics for center and spread.

Once groups have completed their comparative data analysis, use **slides 13–14** to review standard deviation and how it can be used to determine whether the ping pong ball or gummy snack is a more reliable astronaut. Transition to **slide 15** and facilitate a class discussion about identifying the variables in the experiment, stating the problem in statistical terms, and determining which “astronaut” each group has chosen as the most consistent, based on their collected data.

Teacher's Note: Sample Response

Explanatory Variable: Pull-back Angle

Response Variable: Actual Distance

Fixed Variable: Launch Angle

State the Problem: What is the relationship between the distance traveled of an astronaut and the pull-back angle of the catapult?

Most consistent astronaut: Groups determine.*

Use **slide 15** to guide students through the process of determining the best line of regression for a given dataset. Demonstrate how to use statistical tools, such as graphing calculators or software, to analyze their data and assess the strength of their regression models.

Teacher's Note: Using technology to assess the strength of a regression model

While these videos explain how to calculate the linear regression on each device, encourage students to use the same steps to assess Quadratic, Cubic, and Exponential regression to ensure that they choose the regression with the r-squared value that is closest to 1.

Line of Best Fit | Regressions in Desmos | Linear, Quadratic, Cubic, and Exponential

Finding The Correlation Coefficient and Line of Best Fit Using the TI 84

How to Perform Linear Regression with TI nspire

Using Google Sheets for Linear Regression

35 minutes

Extend

Teacher's Note: Preparing for Data Analysis

Before this portion of the lesson, determine whether students will use TI graphing calculators, Google Sheets, or another graphing technology to input and analyze their data.

If using the Spreadsheet, download and share the attached spreadsheet so students can input their data. Duplicate the "Class Data" tab so that there is one for each group in the class and rename each tab to be unique to each group. In their tab, students will record their pull-back angle and distance in the table and the spreadsheet will auto-populate a scatter plot, predicted distance, error, and standard deviation. Review the Example tab for a completed trial with data to see how the spreadsheet functions.

If using calculators, ensure that students are familiar with entering data, generating regression equations, and interpreting statistical outputs. If needed, review with students again, how to determine the best line of regression with their technology.

Determining line of regression in Google Sheets In your data collection software, record the Launch Angle, pull-back angle, and Distance.

1. Edit the Chart. Select the Customize tab. Under the Series Tab, select the Trendline checkbox.
2. Select an appropriate type, and under Label, select the "Use Equation Drop box."
3. Select the Show R^2 checkbox.

Teacher's Note: Data Collection

Students will collect bivariate data and make three models. Each model will consist of 15 trials with a fixed launch angle and paper plate target distance. Within these 15 trials, students will adjust the pull-back angle according to their observations to increase the chance that the astronaut will successfully land on the paper plate. At some point across the domain, they will likely have launch failures or inconsistent ranges. We will remove these outliers and restrict our model to the useful domain. For each launch angle, students will gather approximately 4 data points at varying pull-back angles. Between models, conditions such as the launch angle and paper plate distance can be adjusted as long as they are not adjusted within the 15 trials of that model.

Using **slide 17**, instruct students to continue working in groups to determine the optimal pull-back and launch angles to land their "astronaut" on the paper plate target set 1 to 3 meters from the catapult. Students will use the Extend portion of their Mission Log handout (page 4) to record data. Encourage students to use tape to secure the plate to the floor to prevent migrations between trials.

Teacher's Note: Facilitating Experimentation

This phase will involve trial and error as students collect additional data. Encourage students to experiment with different launch angles, as some angles will have a very narrow range or produce inconsistent launches. Consider asking probing questions such as:

- Did you find any launch angles that produced models with a narrow range or inconsistent results?
- Are there angles that produced models where different pull-back levels yield overlapping results?

Some models may be more consistent at certain ranges and worse at others. Encourage groups to experiment with the distance of the paper plate and help them understand that they may end up with two models to cover the range.

For data collection, groups will conduct 15 additional trial launches at a fixed launch angle. Within those trials, they will adjust the pull-back angle to find the useful domain of their model. At some point across the domain, they will likely have launch failures or inconsistent ranges. We will remove these outliers and restrict our model to the useful domain. For each launch angle, students will gather approximately 4 data points at varying pull-back angles.

Once a group determines two launch angles to cover the 1–3-meter range and produce consistent landings, move to **slide 18**. One of the group members who was previously assigned to measuring will begin inputting the new data into the spreadsheet or graphing calculator. Students will use these tools to:

1. Identify the best regression model for each of their data sets.
2. Observe the direction, form, and strength of the scatterplot between pull-back angle and distance traveled.
3. Evaluate how well their regression model predicts future launches.

20 minutes

Evaluate

Move to **slide 19** and point students to the Evaluate section of their Mission Log handout (page 5). Ask each group to Use Part A to identify their regression model and r-squared value and describe which launch settings yield the most consistent results.

Transition to **slide 20** and ask students to determine which regression model they think would be best to use if their target were 1.75 meters away from their launch point. Explain to students that they will have 5 minutes to select the regression model and use it to determine the settings for their catapult, such as the pull-back and launch angles.

Once the timer ends, move to **slide 21** and instruct the students to place their paper plate target 1.75 meters from their catapult. Students will then begin launching their astronauts, noting the launch and pull-back angle for each attempt. Groups should carefully make adjustments as necessary to the launch pull-back and angle between trials if their astronauts are not landing on the paper plate.

When a group successfully lands their astronaut on the paper plate, they will record the number of attempts it took to achieve their first success in Part B of their handout and the launch and pull-back angle of the successful attempt in Part C.

After identifying a successful launch angle and pull-back angle, students will conduct 10 additional launches using these settings and record how many of the 10 attempts successfully land on the paper plate in Part D of their handout.

As groups finish, use already established classroom procedures to collect student handouts to evaluate.

Resources

- Hamilton, N. (n.d.). Finding the correlation and line of best fit using the TI 84. YouTube. https://www.youtube.com/watch?v=6OGJT1j4L4E
- Home Science Tools. (n.d.). Ping pong catapult. Lesson plans. https://www.sciencebuddies.org/teacher-resources/lesson-plans/ping-pong-catapult
- Lab Hamster. (n.d.). Using google sheets for linear regression. YouTube. https://www.youtube.com/watch?v=yZXNS5AxmIY
- Mr. H. Here to Help. (n.d.). Line of best fit/Regressions in Desmos/Linear, Quadratic, Cubic, and Exponential. YouTube. https://www.youtube.com/watch?v=gy7I9Sj4DnY&t=278s
- Science Buddies. (n.d.). Catapult FAQs. https://www.sciencebuddies.org/science-fair-projects/project_ideas/FAQ_Catapult.shtml
- TI Australia. (n.d.). How to perform linear regression with TI nspire. YouTube. https://www.youtube.com/watch?v=Xse7Aqb0fLI