Chain Rule Explanation

This handout is a step-by-step explanation of the chain rule. The purpose of providing this information is to allow teachers and students to have a scaffolding for making sense of which parts of the derivative are *u* and which parts are *u’.*

1. Find the derivative of $y=\sqrt{1-x^{2}}$

$$y=\sqrt{1-x^{2}}$$

First, let $y=f(u)$

**Notice that this function is a composition of two functions:** $f\left(u\right)=\sqrt{u}$**, where** $u=1-x^{2}$

The derivative of a composite function $y=f(u)$ is $y^{'}=f'(u)∙u'$

**If**$ y=f\left(u\right), then$$f\left(u\right)=\sqrt{u}$

**Therefore, by the chain rule,** $f^{'}\left(u\right)=\frac{1}{2}\left(u\right)^{-\frac{1}{2}}∙u'$

Because $u=1-x^{2}, $then $u^{'}=-2x$

Substituting $u$ and $u^{'}$into the derivative, we get $y^{'}=\frac{1}{2}\left(1-x^{2}\right)^{-\frac{1}{2}}∙(-2x)$

Leaving this derivative unsimplified, we can now clearly label the parts of the derivative with $u $and $u'$.

$y^{'}=\frac{1}{2}\left(1-x^{2}\right)^{-\frac{1}{2}}∙(-2x)$

 u u’