CHAIN RULE EXPLANATION

This handout is a step-by-step explanation of the chain rule. The purpose of providing this information is to allow teachers and students to have a scaffolding for making sense of which parts of the derivative are *u* and which parts are *u'*.

1. Find the derivative of $y = \sqrt{1 - x^2}$

$$y = \sqrt{1 - x^2}$$

First, let $y = f(u)$

Notice that this function is a composition of two functions: $\boldsymbol{f(u) = \sqrt{u}}$, where $\boldsymbol{u = 1 - x^2}$

The derivative of a composite function $y = f(u)$ is $y' = f'(u) \cdot u'$

If $\boldsymbol{y = f(u), then\ f(u) = \sqrt{u}}$

Therefore, by the chain rule, $\boldsymbol{f'(u) = \frac{1}{2}(u)^{-\frac{1}{2}} \cdot u'}$

Because $u = 1 - x^2$, then $u' = -2x$

Substituting u and u' into the derivative, we get $y' = \frac{1}{2}(1 - x^2)^{-\frac{1}{2}} \cdot (-2x)$

Leaving this derivative unsimplified, we can now clearly label the parts of the derivative with u and u'.

$$y' = \frac{1}{2}\underset{\underset{u}{\uparrow}}{(1 - x^2)}^{-\frac{1}{2}} \cdot \underset{\underset{u'}{\uparrow}}{(-2x)}$$