I Sub, U-Sub, We All Sub

Integration: U-Subtitution, Part 1

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Published by *K20 Center*

Grade Level	12th Grade	**Time Frame**	75 minutes
Subject	Mathematics	**Duration**	1-2 class period(s)
Course	AP Calculus		

Essential Question

How can we undo the chain rule? In other words, how can we find the antiderivative of a function that has been differentiated using the chain rule?

Summary

This lesson builds on students' prior knowledge of basic integration techniques in calculus. The purpose of this lesson is to allow students to explore how to "undo" a function's derivative that was found using the chain rule. The chain rule is a derivative technique that allows students to find derivatives of composite functions.

Snapshot

Engage

Begin by posing the question, "How do we find the derivative of the function f(x)=sqrt(1-x^2)," then following with, "How can we undo this derivative?"

Explore

The exploration allows students a chance to make better sense of and explore how to undo some simple chain rule examples, with the goal of students exploring what works and what does not work.

Explain

Students and the teacher work together to create an Anchor Chart for what worked in the Explore activity and what did not work.

Extend

Students begin to label parts of the functions that didn't work in order to understand how the process of substitution works.

Evaluate

Students use the What? So What? Now What? instructional strategy to reflect on their learning from the lesson.

Standards

Oklahoma Academic Standards for Mathematics (Process Standards)

M.5: Develop a Productive Mathematical Disposition: Hold the belief that mathematics is sensible, useful and worthwhile. Students will develop the habit of looking for and making use of patterns and mathematical structures. They will persevere and become resilient, effective problem solvers.
M.6: Develop the Ability to Make Conjectures, Model, and Generalize: Make predictions and conjectures and draw conclusions throughout the problem solving process based on patterns and the repeated structures in mathematics. Students will create, identify, and extend patterns as a strategy for solving and making sense of problems.

Attachments

- Chain Rule Explanation - Spanish.docx
- Chain Rule Explanation - Spanish.pdf
- Chain Rule Explanation.docx
- Chain Rule Explanation.pdf
- Engage Questions - Spanish.docx
- Engage Questions - Spanish.pdf
- Engage Questions.docx
- Engage Questions.pdf
- Exploring What Works - Spanish.docx
- Exploring What Works - Spanish.pdf
- Exploring What Works.docx
- Exploring What Works.pdf

Materials

- Explore activity
- Extend problems

Engage

To begin this lesson, display the equation of your choice from the *Engage Questions* attachment.

The Choice Is Yours

There are several equations in the attachment. The goal is not to do all of them, but to choose one based on where your students are in their sequence. For example, if you have covered derivative rules involving trig functions, you might choose to use the trig equation. Go with what you know!

Ask students to find the derivative of the equation that you chose using the chain rule. Students can work individually or with a partner.

What's the Goal? Process Is Key

Remember to keep the big picture in mind when doing this Engage activity. The goal is to eventually "undo" derivatives involving the chain rule. It is important for students to be able to understand the steps involved in finding the derivative and to be able to articulate what is happening in the process.

Explore

Explaining the Chain Rule Using U and U' or U and Du

In the attachments, refer to the *Chain Rule Explanation* handout, but do not distribute the handout to students. The purpose of the handout is for you to use as a reference as you work with individual groups to help them identify which part of the derivative is *u* and which part is *du* or u'.

In this activity, students will be asked to explore simple integrals in order to determine what "works" and what "doesn't work."

Distribute the Exploring What Works handout to each student. Have students work with a partner to complete the task.

So, What Works?

By asking "what works," we are simply asking which integrals can be found using existing antiderivative techniques.

Eagle Eye!

Keep a close eye on students and monitor their progress. If students are getting stuck, assist them without directly giving the answers. Try to make sure that students are able to see what is working and what is not working and why.

Change It Up

If you don't like the problems prescribed in the activity, feel free to change them. The problems listed for students to solve assume that students know how to find antiderivatives of many different types of functions.

Explain

Why Is This Going So Slow?

Okay, I know it is going slow up to this point, but that is for good reason. The purpose here is for students to begin to see that the reason that some of the antiderivatives don't happen to work as naturally as others. The goal of the anchor chart is for students to articulate why some antiderivatives work and others don't (YET).

Creating an Anchor Chart

1. Using a whiteboard, oversize sticky note, document camera, or interactive whiteboard, create a T-chart by dividing the display into two halves.
2. Title one half "Worked" and the other half "Didn't Work."
3. Ask students to categorize the functions into the two categories.

Pose the question, 'Why did these work and those did not?' Using the Think-Pair-Share instructional strategy, give students a few minutes to think to themselves and then discuss with a partner. Finally, solicit a few responses from pairs to write on the anchor chart for the entire class to see.

Extend

Ask students to retrieve their *Chain Rule Explanation* handout from the Explore activity. Have students work in pairs.

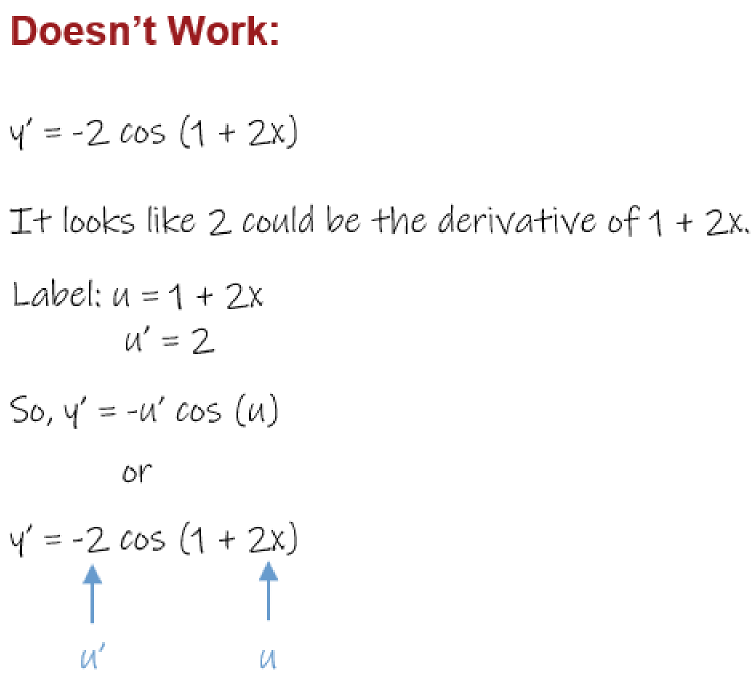
In the blank space at the bottom of the handout, ask students to write out the first problem that "didn't work."

Using the Think-Pair-Share strategy, pose the following question: "Is there part of this equation that looks like it might be the derivative of another part?"

Give students adequate time to think through this question, allow them to speak with their partner after a minute or two, and then solicit responses.

Identifying U and U'

The purpose of this question is for students to begin wrestling with the idea of seeing the function's components, u and u'. Make sure to give students ample time to see this. It might seem like it is taking forever, but once students can see that the "inside" of the function is u (the part in parentheses) and the derivative u' is located at the end of the function (multiplied on the outside of the parentheses), then you're in good shape.



The first equation that didn't work is above. The goal is to give students space to label parts of the equation that look like derivatives of the other parts of the equation. The parts might not be perfect, and that's okay.

Repeat this process with the remaining three equations that "didn't work."

Evaluate

Have students reflect on what they've done using the What? So What? Now What? strategy.

- What? In a few sentences, have students describe what they did today.
- So what? Ask students to explain the importance of what they did today.
- Now what? Ask student to hypothesize how they will use this information in the future.

Collect responses and share a few with the class at the start of the next day.

Resources

- K20 Center. (n.d.). Think-pair-share. Strategies. https://learn.k20center.ou.edu/strategy/d9908066f654727934df7bf4f5064b49
- K20 Center. (n.d.). What? So what? Now what? Strategies. https://learn.k20center.ou.edu/strategy/b30762a7557ba0b391f207f4c6002113