I Sub, U-Sub, We All Sub Again

Integration: U-Subsitution, Part 2

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Published by *K20 Center*

Grade Level	12th Grade	**Time Frame**	75 minutes
Subject	Mathematics	**Duration**	2-3 class periods
Course	AP Calculus		

Essential Question

How can we undo the chain rule? In other words, how can we find the antiderivative of a function that has been differentiated using the chain rule?

Summary

This lesson is a follow-up to I Sub, U-Sub, We All Sub. In the previous lesson, students worked with the teacher to create an anchor chart of what "worked" and what "didn't work" for anti-differentiating derivatives of functions. The goal of this lesson is to build on that knowledge in order for students to begin seeing that what "worked" worked for a specific reason. Coupling this notion with the connection to what "didn't work" will allow students to see how U-substitution is a manipulated form of what did work in Part 1.

Snapshot

Engage

Using the anchor chart from the I Sub, U-Sub, We All Sub lesson, teachers and students identify what worked and what didn't work. Students label part of the equation as u and the other part as du.

Explore

Students explore the idea that part of the derivative of the chain rule looks similar to the derivative of another part of the function. Students then explore how to identify these parts as u and du.

Explain

Clarify what u represents and what du represents. Students then articulate that the integral of u du, looks like the integral of x dx.

Extend

Practice with these new types of problems in a scavenger hunt, identifying u and du, and then finding the antiderivative.

Evaluate

What? So What? Now What?

Standards

Oklahoma Academic Standards for Mathematics (Process Standards)

PK.S: Mathematical Actions and Processes
M.1: Develop a Deep and Flexible Conceptual Understanding: Demonstrate a deep and flexible conceptual understanding of mathematical concepts, operations, and relations while making mathematical and real-world connections. Students will develop an understanding of how and when to apply and use the mathematics they know to solve problems.
M.2: Develop Accurate and Appropriate Procedural Fluency: Learn efficient procedures and algorithms for computations and repeated processes based on a strong sense of numbers. Develop fluency in addition, subtraction, multiplication, and division of numbers and expressions. Students will generate a sophisticated understanding of the development and application of algorithms and procedures.
M.3: Develop Strategies for Problem Solving: Analyze the parts of complex mathematical tasks and identify entry points to begin the search for a solution. Students will select from a variety of problem solving strategies and use corresponding multiple representations (verbal, physical, symbolic, pictorial, graphical, tabular) when appropriate. They will pursue solutions to various tasks from real-world situations and applications that are often interdisciplinary in nature. They will find methods to verify their answers in context and will always question the reasonableness of solutions.
M.4: Develop Mathematical Reasoning: Explore and communicate a variety of reasoning strategies to think through problems. Students will apply their logic to critique the thinking and strategies of others to develop and evaluate mathematical arguments, including making arguments and counterarguments and making connections to other contexts.
M.5: Develop a Productive Mathematical Disposition: Hold the belief that mathematics is sensible, useful and worthwhile. Students will develop the habit of looking for and making use of patterns and mathematical structures. They will persevere and become resilient, effective problem solvers.
M.6: Develop the Ability to Make Conjectures, Model, and Generalize: Make predictions and conjectures and draw conclusions throughout the problem solving process based on patterns and the repeated structures in mathematics. Students will create, identify, and extend patterns as a strategy for solving and making sense of problems.
K.PS2: Develop the Ability to Communicate Mathematically: Students will discuss, write, read, interpret and translate ideas and concepts mathematically. As they progress, students' ability to communicate mathematically will include their increased use of mathematical language and terms and analysis of mathematical definitions.

Attachments

- Evaluate - Spanish.docx
- Evaluate - Spanish.pdf
- Evaluate.docx
- Evaluate.pdf
- Explain Handout - Spanish.docx
- Explain Handout - Spanish.pdf
- Explain Handout.docx
- Explain Handout.pdf
- Exploring U-Sub - Spanish.docx
- Exploring U-Sub - Spanish.pdf
- Exploring U-Sub.docx
- Exploring U-Sub.pdf
- Scavenger Hunt - Spanish.docx
- Scavenger Hunt - Spanish.pdf
- Scavenger Hunt.docx
- Scavenger Hunt.pdf

Materials

- Paper and pencil

Engage

Display the anchor chart from the I Sub, U-Sub, We All Sub lesson.

U and Du

The biggest takeaway from the previous lesson, I Sub, U-Sub, We All Sub, is that part of the equation looks like the derivative of the other. Where students labeled u and u', the goal today is to replace u' with du, which represents the derivative of u.

Have students articulate which parts of the equation look like the derivative of the other part (u and u').

Pose the question, "What if we replace u' with du? Would that change anything?"

On the anchor chart, replace u' with du in each of the equations that "didn't work."

Teacher-Directed or Student-Directed?

The last step for the anchor chart can be done by the teacher or the students. Depending on your comfort level, feel free to make the changes yourself or ask students to volunteer.

Explore

Distribute the *Exploring U-Sub* handout to each student. Ask students to work in pairs to go through the problems.

As students progress through the activity, check in with each pair to help clarify questions that they might have.

Explain

After students have finished both parts of the Explore activity, move on to the Explain activity, which is intended to clarify misconceptions and make sure students all understand the concepts.

I Know What You're Thinking...

When are we ever going to get to integrals? We're here. This is where students have the opportunity to connect their prior learning of basic integral forms to u-sub.

Project or print the questions in the *Explain Handout* and ask students to explain each question. Give students ample time to work with their partner to explain their reasoning.

Extend

Have students complete the scavenger hunt, either individually or in pairs. Be sure to make yourself available if students have questions.

Scavenger Hunt

You'll find the Scavenger Hunt document in the attachments. Each page contains a problem to be solved and a solution. To set up the task, hang all posters around the room (and in the hall if you are comfortable). Ask students to start wherever they like. The equation on the bottom is a problem to be solved. Above the line is the solution to a different problem. Once students solve the problem below the line, they are to move about the room until they find the solution for their problem. After finding the solution, they solve the next problem, which is below the line on the same page. Students continue this process until they are back to where they started.

From Personal Experience: Don't Be Too Tricky

As fun as it is to mess with students and post the pages behind doors, between pages in books, or on the ceiling, the goal is to learn mathematics, not for students to hone their detective skills. Mix up the pages and tape them around the room, but make sure they are visible to the naked eye.

Evaluate

Display the attached *Evaluate* document to have students reflect on their learning and try a challenge problem.

Give students two or three minutes each to complete the first two questions. Allow students to explore the final question and use it as a starting point for conversations about manipulating the substitution.

Everything Is So Neat

You've probably noticed that nearly all of the substitutions are very neat and tidy. Students rarely, if ever, have to manipulate du in order to substitute it. The challenge problem at the end will cause students to have to think deeply about their substitution and how they can make it work. This problem is a great conversation starter and transition into more difficult problems in the future.

And...we're done!

Resources

- K20 Center. (n.d.). I sub, u-sub, we all sub? Lessons. https://learn.k20center.ou.edu/lesson/b30762a7557ba0b391f207f4c600cc36