

Pythagor-eatin' Theorem

Constructing the Pythagorean Theorem

Cacey Wells, Lindsay Hawkins
Published by *K20 Center*

This work is licensed under a Creative Commons CC BY-SA 4.0 License

Grade Level	7th – 8th Grade	**Time Frame**	3 class periods
Subject	Mathematics	**Duration**	115 minutes
Course	Middle School Mathematics		

Essential Question

What is the Pythagorean theorem, and does it work for all right triangles?

Summary

This lesson engages students in constructing the Pythagorean theorem using Cheez-It® crackers. Students will not only be able to visually see the theorem, but they will also use the Pythagorean theorem algebraically to find the missing sides of right triangles.

Snapshot

Engage

Students write a Preflection on what they know about right triangles.

Explore

Students use Cheez-It® crackers to construct the Pythagorean theorem.

Explain

Students use their observations to propose the relationship between the sides of a right triangle, then formalize their understanding of the Pythagorean theorem.

Extend

Students apply their understanding of the Pythagorean theorem to find the lengths of the missing sides of given triangles.

Evaluate

Students compare and contrast their Preflection with their learning experiences.

Standards

ACT College and Career Readiness Standards - Mathematics (6-12)

G404: Find the length of the hypotenuse of a right triangle when only very simple computation is involved (e.g., 3-4-5 and 6-8-10 triangles)
G508: Given the length of two sides of a right triangle, find the third when the lengths are Pythagorean triples
G602: Use the Pythagorean theorem

Oklahoma Academic Standards Mathematics (8th Grade)

PA.GM.1.1: Justify the Pythagorean theorem using measurements, diagrams, or dynamic software to solve problems in two dimensions involving right triangles.

Attachments

- Before and After Thoughts—Pythagor-eatin' Theorem - Spanish.docx
- Before and After Thoughts—Pythagor-eatin' Theorem - Spanish.pdf
- Before and After Thoughts—Pythagor-eatin' Theorem.docx
- Before and After Thoughts—Pythagor-eatin' Theorem.pdf
- Lesson Slides—Pythagor-eatin' Theorem.pptx
- Paper Crackers—Pythagor-eatin' Theorem.docx
- Paper Crackers—Pythagor-eatin' Theorem.pdf
- Pythagorean Theorem Trifold—Pythagor-eatin Theorem - Spanish.docx
- Pythagorean Theorem Trifold—Pythagor-eatin Theorem - Spanish.pdf
- Pythagorean Theorem Trifold—Pythagor-eatin Theorem.docx
- Pythagorean Theorem Trifold—Pythagor-eatin Theorem.pdf
- Right Triangle Cutouts—Pythagor-eatin' Theorem.docx
- Right Triangle Cutouts—Pythagor-eatin' Theorem.pdf
- Right on Target—Pythagor-eatin' Theorem.docx
- Right on Target—Pythagor-eatin' Theorem.pdf

Materials

- Lesson Slides (attached)
- Before and After Thoughts handout (attached; one per student; print one-sided)
- Right on Target handout (attached; one per small group; print one-sided on 11”x17” paper)
- Right Triangle Cutouts handout (optional; attached; print one-sided)
- Pythagorean Theorem Trifold handout (attached; one per student; print two-sided)
- Envelopes (one per student)
- Standard-size Cheez-It® (or similar brand) crackers (at least 50 unbroken crackers per small group)
- Paper Crackers handout (optional; attached; print one-sided)
- Pens or pencils
- Paper
- Card stock or construction paper (11”x17”; optional; one per student)

Preparation

During the Explore phase of this lesson, small groups of 2–3 students will investigate the Pythagorean theorem with Cheez-It® crackers. Each group will need at least 50 unbroken crackers. Consider preparing for this activity by putting 50 crackers in a zip-top bag so that you have enough bags for each group to have one.

If you would prefer not to use food or for students with a gluten allergy, consider printing the attached **Paper Crackers** handout and cutting out the 1-inch cracker tiles as an alternative. There are 60 cracker tiles on the page. It is strongly recommended that you print this handout on cardstock paper so that the cracker tiles will hold their shape and hold up better over time.

Also, small groups will each need the **Right on Target** handout (printed on 11”x17” paper) or a sheet of 11”x17” cardstock or construction paper for students to use as a placemat for their crackers. If students are using cardstock or construction paper, print the attached **Right Triangle Cutouts** handout and cut out the eight triangles; print enough copies for each small group to have one triangle.

10 minutes

Engage

Introduce the lesson using the attached **Lesson Slides**. Display **slide 3** and share the Preflections strategy. Give each student a copy of the attached **Before and After Thoughts** handout. Here, students are asked to do their best to answer the following questions:

1. If Triangle *DEF* is a right triangle, with angle *F* being the right angle, side *d* = 15, and side *e* = 20, how do you think you could figure out the length of side *f*?
2. Which sides are the ***legs*** and which side is the ***hypotenuse*** of Triangle *DEF*?

Give students approximately 5 minutes to write responses on their handout (this will feel like a long time).

While students are writing, pass out an envelope to each student and monitor progress. If students are struggling, remind them that these are just educated guesses and they will be able to come back to their answers later.

Move to **slide 4** and ask students to ensure their name is on their paper and on their envelope. Have students fold their paper so that it fits within the envelope, place it in the envelope, and seal it shut. Tell students that they will revisit these at the end of the lesson, then collect all envelopes.

After students have completed their Preflections, share **slides 5–6** with the essential question and lesson objectives.

30 minutes

Explore

Display **slide 7** and have students get into small groups of 2–3, or assign small groups. Have students gather materials or distribute about 50 whole Cheez-It® crackers (no broken pieces) and a copy of the attached **Right on Target** handout to each small group. Direct groups to label their triangle: Label the two sides forming the right angle as ***a*** (the shortest side) and ***b***. Label the longest side opposite the right angle ***c***.

Show **slide 8** and guide students to:

1. Using Cheez-It crackers, line the perimeter (outside edge) of the right triangle. *(Note: the crackers should fit almost perfectly along the outside edges.)*
2. Using the rest of the crackers, build squares outward from each side of the triangle. The length of each triangle side becomes the *side length* of the square you build. *(Note: If needed, provide students with an example: "If a side is 7 Cheez-Its long, then build a 7 by 7 square along that edge of the triangle.* Cheez-It *crackers will be touching but not overlapping.")*

Use the hidden **slide 9** as a visual guide for steps 1–2. Then transition to **slide 10** and guide the students to:

3. Once the squares are built on all sides of the right triangle, take the Cheez-Its from the two sides forming the right angle (sides *a* and *b*) and begin stacking those on top of the Cheez-It crackers that form the square on the longest side, opposite the right angle (side *c*). *(Note: Stack only one Cheez-It on top of each of the existing Cheez-It crackers on side c.)*

Use the hidden **slide 11** as a visual guide for step 3.

35 minutes

Explain

While groups still have their Cheez-It® activity on their desks, show **slide 12**. Give each student a copy of the attached **Pythagorean Theorem Trifold** handout. Direct students' attention to the "Pythagorean Theorem" section and have them label the triangle with *a*, *b*, and *c*, in the same manner as they did for the previous activity. Then ask students to discuss within their groups what they think the relationship between a^2, b^2, and c^2 is. If students struggle, ask them to think about the activity they just completed and how a^2 was represented with the Cheez-It crackers.

After a couple of minutes of discussion, transition to **slide 13** and ask for volunteers to come to the board (to the slide) to write their responses.

Possible Response to the Relationship Between the Triangle's Sides:

Student answers will vary. Keep in mind that at this stage, the relationship they write may be mostly verbal and may be partially pictorial or algebraic: "When you combine the squares for sides a and b (add a^2 and b^2), they equal the square for side c (c^2)." Students are asked later to represent the theorem algebraically, so do not yet require an algebraic representation.

Display **slide 14** and share the pictorial representation on the slide (referencing their Cheez-It crackers) and ask the class how they could use a^2, b^2, and c^2 to write an equation to describe the relationship between those variables and represent the Pythagorean theorem algebraically. Facilitate a whole-class discussion, then transition to **slide 15** to reveal: $a^2 + b^2 = c^2$.

Now that students have made the connection between the visual and algebraic representations of the Pythagorean theorem, have students clean up their Cheez-It crackers. Encourage students to use their Right on Target handout as a tool to catch cracker crumbs.

Teacher's Note: Pacing the Lesson

During a traditional 45-minute class period, this lesson will naturally need to pause and resume the next day. A smooth transition could be to pause the lesson here, after students have cleaned up their Cheez-It activity, and resume the lesson by introducing vocabulary and then finishing the trifold. Have students store their trifold in a place where they can easily find it to later complete.

After students have cleaned up their space from their Cheez-It® activity, direct their attention to the "Vocabulary and Symbols" section of their trifold. Ask them to use the information on the slides to describe the vocabulary terms in their own words and draw the symbol, if there is one associated with it. Encourage them to sketch a picture if they find it helpful for understanding or remembering the vocabulary word. Transition through **slides 16–18** to review or introduce: *right triangle*, *legs of a triangle*, and *hypotenuse*. Use the hidden **slide 17** if you find that students need a reminder for *angles* and *triangles*.

Pause on **slide 18** and ask the class which word they should use to describe the side they earlier labeled "*c*." Repeat this question for sides "*a*" and "*b*," then reveal the accurate vocabulary on **slide 19** and have students complete the triangle they started labeling earlier on the front of their trifold. Remind students to also make notes in the "Vocabulary and Symbols" section, as needed. For example, students may want to write the Pythagorean theorem in words or algebraically in the "Vocabulary and Symbols" section of their trifold.

Show **slide 20** and direct students' attention to the "Am I Right?" section of the trifold. Remind students that they just learned that the Pythagorean theorem works for any right triangle, and then ask them the rhetorical question, "do you think it works for any triangle?" Have students work in pairs and use the given side lengths to determine if $a^2 + b^2 = c^2$ and if those side lengths form a right triangle for the first question.

After a couple of minutes, move to **slide 21** and ask for volunteers to share their thinking. Students should have concluded that the Pythagorean theorem worked for the first question (so it is a right triangle), as this triangle had side lengths of 6, 8, and 10 feet.

Teacher's Note: Purpose

Here, the goal is for students to see the connection between the Pythagorean theorem and right triangles.

Repeat this with Question 2 using **slides 22–23**. Here, students should conclude that the Pythagorean theorem does not work, since the given triangle is not a right triangle having side lengths of 7, 8, and 12 inches.

Repeat this again with Question 3 using **slides 24–25**. Here, students should conclude that the Pythagorean theorem does work, since the given triangle has side lengths of 15, 12, and 9 meters. If students struggle with this question, ask more questions than you give answers. Ask which length they used for the hypotenuse and what the definition of the hypotenuse is. Students may forget that it is the longest side and just substitute the numbers in for a, b, and c in the order they are given.

Once students complete the three questions, summarize that the Pythagorean theorem ***only*** works for right triangles.

Display **slide 26** and have pairs look for a pattern or what they notice is similar between the triangles from Question 1, Question 3, and the triangle from their Cheez-It activity. After giving time for pairs to discuss, bring the class together for a whole-class discussion. As volunteers share that the side lengths are multiples of each other, share with the class that these (3–4–5, 6–8–10, and 9–12–15) are examples of *Pythagorean triples*.

Sample Student Responses:

- Questions 1 and 3 have a proportional relationship to the side lengths of the Cheez-It triangle.
- Side a is $3 \times 2 = 6$; side b is $4 \times 2 = 8$; side c is $5 \times 2 = 10$.
- The sides of Question 3 are three times larger than each side of the 3, 4, 5 triangle.
- Does the proportional relationship work with all right triangles?

If pausing the lesson here, have students put their trifold in a place where they can easily find it for the next phase of the lesson.

30 minutes

Extend

Show **slide 27** and resume the "Vocabulary and Symbols" section. Introduce examples of perfect squares: 9, 16, 25, etc. Ask the class why they think those numbers are called perfect squares. If students struggle to respond, ask them to think about where they saw 9, 16, and 25 during the Cheez-It® activity. Help students make the connection that these perfect square numbers can be represented geometrically with squares.

Move to **slide 28** and introduce the last vocabulary word: *square root symbol (√)*.

Have students get a calculator—follow classroom norms for this. Display **slide 29** and direct students' attention to the "What's My Hypotenuse?" section of the trifold. Have students work with the same partners as before, or have students find new partners to complete Question 4. Here, students are asked to use the Pythagorean theorem to find the unknown length of the hypotenuse. As pairs work, circulate the room and listen to student discussions. Give guidance where needed, but allow pairs to have a healthy struggle.

When students complete Question 4, move to **slide 30** for students to check their work. Give students time to make corrections and ask questions.

Display **slide 31** and ask the class if they think the side length of a triangle could have a decimal value. Facilitate a brief discussion, then move to **slide 32** and have students begin Question 5.

Sample Student Responses:

- I think you can have a decimal, but what place value do you round it to?
- Maybe if it is a rational number.

As students finish Question 5, move to **slide 33** for students to check their work. Give students time to make corrections and ask questions.

Similarly, direct students' attention to the "What's My Leg Length?" section of the trifold and display **slide 34**. Ask students to begin Question 6. While pairs work, circulate the room. After a couple of minutes, ask the class what is different about this question compared to the previous two questions. Students should notice that this triangle has an unknown leg length (and a given value for the hypotenuse).

As students finish Question 6, move to **slide 35** for students to check their work. Give students time to make corrections and ask questions.

Repeat this for Question 7 using **slides 36–37**.

Now that students have completed writing in their trifold, transition through **slides 38–39** to show students how to fold their trifold along the vertical lines. Then, using the blank backside of this trifold, students can glue or tape the resource into their math notebooks.

10 minutes

Evaluate

Display **slide 40** and distribute students' Preflection envelopes, which they completed during the Engage section, back to each student.

Ask students to individually open their envelopes and read through their earlier writing.

Display **slide 41** and have students answer the following prompts in the “Reflection” section of their handout:

3. What are the main differences you notice between what you learned during the lesson compared to what you initially wrote?
4. What is the Pythagorean theorem, and what is it used for?
5. What are two major takeaways from today's lesson that you will remember?

Resources

- K20 Center. (n.d.). Preflections. Strategies. https://learn.k20center.ou.edu/strategy/191