K20 LEARN

Zombie Takeover

Logistic Functions

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Published by *K20 Center*

Grade Level	10th – 12th Grade	**Time Frame**	80-105 minutes
Subject	Mathematics		
Course	AP Calculus, Algebra 2, Precalculus		

Essential Question

How can logistics functions be used to model real-world phenomena?

Summary

Students will investigate logistic functions as mathematical models of real-world situations, such as zombie outbreaks, and make comparisons with exponential growth models. Students will learn about the properties of the logistic curves and how to graph logistic functions by hand. This lesson can be taught in Algebra 2 (similarly sequenced in a Precalculus course) after students have mastered exponential and logarithmic functions. This lesson can also be used as a review in a Calculus course before students learn about differential equations.

Snapshot

Engage

Students watch a video and read a scenario about the spread of zombie infection and then make a prediction.

Explore

Students simulate a zombie outbreak and gather data.

Explain

Students use an online calculator to generate a logistic function to model their data, then formalize their understanding of logistic functions, and learn how to sketch a logistic curve by hand.

Extend

Students apply what they learned by graphing logistic functions and identifying key characteristics of their graphs.

Evaluate

Students determine if exponential or logistic functions would best model given growth scenarios.

Attachments

- Exit Ticket—Zombie Takeover - Spanish.docx
- Exit Ticket—Zombie Takeover - Spanish.pdf
- Exit Ticket—Zombie Takeover.docx
- Exit Ticket—Zombie Takeover.pdf
- Exploring a Zombie Outbreak—Zombie Takeover - Spanish.docx
- Exploring a Zombie Outbreak—Zombie Takeover - Spanish.pdf
- Exploring a Zombie Outbreak—Zombie Takeover.docx
- Exploring a Zombie Outbreak—Zombie Takeover.pdf
- Lesson Slides—Zombie Takeover.pptx
- Using Key Characteristics—Zombie Takeover - Spanish.docx
- Using Key Characteristics—Zombie Takeover - Spanish.pdf
- Using Key Characteristics—Zombie Takeover.docx
- Using Key Characteristics—Zombie Takeover.pdf

Materials

- Lesson Slides (attached)
- Exploring a Zombie Outbreak handout (attached; one per pair; printed front/back)
- Using Key Characteristics handout (attached; one per student; printed front only)
- Exit Ticket handout (attached; one per student; printed front only)
- Pencils
- Paper
- Student devices with Internet access
- Graphing calculator (optional; one per pair)

15 minutes

Engage

Introduce the lesson using the attached **Lesson Slides**. Display **slide 3** to share the lesson's essential question with students. Go to **slide 4** to share the lesson's learning objectives. Review each of these with students to the extent you feel necessary.

Go to **slide 5** and inform students that they are going to watch a silent video and need to be ready to share what they think the video represents. Show **slide 6** and click the image to view the "Simulation of a Zombie Outbreak!" video. (Clicking the image opens a browser tab to view the video.) After students watch this silent video, ask them to share what they think the video represents.

If students struggle to make the connection to zombies, ask what fictional events or pop culture TV shows that could be represented by the map.

Have students find a partner or assign partners, then read the following prompt on **slide 7**:

When the first human on Earth got infected, no one believed it. We all thought zombies could never be real. As time went by, it was obvious that we had underestimated the swiftness with which human populations could decrease. Our town was one of the last to get hit, but it was brutal. By the time the first infection came, so many residents had left or died of starvation that only 2,000 of us remained to see it.

If there is one zombie among us, how long will it take for all of us to become infected?

Have students work with their partner to write everything they know about zombies and to predict how long it would take to turn a population of 2000 community members into zombies starting with only 1 zombie.

Sample Student Responses

Zombies bite people and those people turn into zombies. I have no idea how to predict the amount of time to change 2000 people, but I am guessing it is not very long considering every single bite results in two zombies. For example, one zombie bites a human, resulting in 2 zombies; 2 zombies bite 2 humans resulting in a total of 4 zombies; and so on: $1 \cdot 2=2$, $2 \cdot 2=4$, $4 \cdot 2=8$, $8 \cdot 2=16$, ...; this continues until there are 2000 zombies. I know that $2^{10} = 1024$, and you can't have a partial bite, so after 11 bites the whole community would be zombies.

Teacher's Note: Guiding the Activity

Some students may have more experience with exponential relationships and give a formula like $y = 2^x$ and then set $y = 2000$ and try to solve for x.

It is also okay for students to be unsure about this very-open-ended question. At this time, you want to see students thinking about what they need to know and creating a plan for how they might solve this problem.

Have students share their responses and explain how they found their solution. Be sure to encourage the variety of approaches. At this time, do not correct or hint to students that they are incorrect. Use student responses to determine if students need a quick refresh on exponential growth.

20 minutes

Explore

Teacher's Note: Using the Desmos Graphing Calculator

Try to become familiar ahead of time with the Desmos Studio graphing calculator to better help students navigate through the following activity. It is not necessary for students to have Desmos accounts to use the Desmos Studio graphing calculator. For more information, go to https://k20center.ou.edu/externalapps/graphing-calculator.

Optional Preparation

The following activity uses Desmos as the graphing calculator. If you have a specific graphing calculator that you would prefer your students use, update slides 11–12 to reflect the directions needed for your specific tool.

Show **slide 8** and read the following prompt:

Our town quickly turned from a safe hiding space into the worst place to try to survive. With zombie populations increasing and resources depleting, several of us decided to run. We spent days hiding out in empty grocery stores and vacated homes to plan our escape, and we were able to make it out of town to a remote cabin that a family had abandoned. By the time our plan was executed, there were only 20 of us left.

Show **slide 9** and continue reading the prompt below:

We've been safe here for a while. I'm a little worried about Joe, though—he's been extremely tired ever since we arrived. He said he wasn't feeling well. What if he has the zombie infection and doesn't know it yet?

How long will it take for all of us to become infected? Let's run a simulation and find out!

Show **slide 10** and preview the activity with the students. Imagine a bag of red and blue tiles where the red tiles represent the zombies and the blue tiles represent the humans. The total population is 20. Draw two tiles from the bag, representing a random interaction between two community members. It could be 2 humans (2 blue tiles), 2 zombies (2 red tiles), or 1 human and 1 zombie (1 blue and 1 red tile) interacting.

Display **slide 11** and explain the procedure for what to do with the interactions:

- If you draw 2 humans (2 blue tiles) or 2 zombies (2 red tiles)—if two of the same types of community members interact—then record your result, (the digital simulation will automatically place the tiles back in the bag as long as "with replacement" is still selected), and repeat the drawing.
- If you draw 1 human (1 blue tile) and 1 zombie (1 red tile)—if a human interacts with a zombie—then record your result, remove the human (blue tile) and replace it with a zombie (red tile), and repeat the drawing.

Now that students have a sense of how the simulation works, pass out one copy of the attached **Exploring a Zombie Outbreak** handout to each pair of students and review the directions on the handout about how to record their results. Show **slide 12**, and direct students to go to https://bit.ly/3LEgtbh. Here, students are using the CPM Probability Generators, not to calculate probability, but to run their zombie outbreak simulation. On the left, students have a quick-reference to the directions for how the simulation tool works. Guide students to make sure that the number of red tiles is 1 and the number of blue tiles is 19 for their first interaction (drawing of tiles) by right-clicking on the bag.

Display **slide 13** and continue explaining the process of the simulation. Direct students to work with their partner and start their simulations, recording their results on their handout. As students begin working, remind them to check that the number of humans plus the number of zombies always equals 20.

Teacher's Note: Guiding the Activity

Students may ask how many interactions or how many rows of their table they will need to complete. That is not a definite number, as it just depends on how their interactions go. For example, they may have only one zombie for the first dozen interactions, then it will stay at two zombies for a while or quickly change to three zombies. Every group should have a different table of data. Regardless, the handout should have enough rows, and it is very unlikely that they will need more. If they need more rows, have students use a piece of notebook paper.

Here is a sample set of data and the corresponding regression function in Desmos: Zombie Outbreak - Population 20.

Once students complete their simulation, show **slide 14**. Direct students to go to desmos.com, click "Graphing Calculator," add a table, and enter their data into the table.

15 minutes

Explain

Ask students to compare this trend of data to what they know about exponential growth: *How are they the same? How are they different?*

Sample Student Responses

Both exponential graphs and these data points look the same for the first interval of time, but our data does not keep growing. It kind of levels off.

Now ask students what they think caused the data to not continuously grow.

Sample Student Responses

There are not infinitely many humans, so once everyone is a zombie, there is no more growth.

The more zombies there were, the less likely it was to have a zombie interact with a human, so the infection rate was slowing down.

Display **slide 15** and use this slide to direct students on how to generate a curve that models their data.

Display **slide 16** and explain that the graph on their screen is a logistic curve with two asymptotes. Clarify to students that the asymptotes on the slide are not visible in Desmos because they represent where the function is approaching but not what the function equals. Facilitate a discussion about where the asymptotes are and why they are at $y = 0$ and $y = 20$. Direct students' attention to the maximum growth point and explain this vocabulary term. In Calculus, this point is known as the point of inflection, where the graph changes concavity. Discuss the characteristics of the graph and parameters of the function to the extent you see necessary.

Show **slide 17** and review the procedure for graphing a logistic function by hand.

Teacher's Note: Guiding the Lesson

It is not encouraged that students remember the formula to find the ordered pair of the maximum growth point. Instead, help students see why it has a y-value that is half of the carrying capacity. If students know the y-value, then they can algebraically solve for x. The approach for finding the maximum growth point may differ in a Calculus course.

20 minutes

Extend

Teacher's Note: Preparation

Decide whether you want the Using Key Characteristics handout to be guided practice or independent practice. The sample responses to the handout are on hidden slides, so if you would like the class to check their work as they go, unhide slides 18–21.

Display **slide 18** and pass out a copy of the attached **Using Key Characteristics** handout to each pair of students. Instruct students to work with their partner to graph the first two equations and label the key characteristics of the graph. For questions 3–4, students are provided a graph and asked to write what they know about the graph. Encourage students to use academic vocabulary.

As time allows, ask for volunteers to explain the process and reasoning.

Optional Slides

Unhide and display **slides 19–22** so students can check their work. Ask for volunteers to explain their work.

10 minutes

Evaluate

Use the Exit Ticket strategy to individually assess what students have learned from the lesson. Show **slide 23** and pass out a copy of the attached **Exit Ticket** handout to each student. Students are asked to read a scenario and decide if an exponential or logistic function would best model the growth described in the scenario.

After students have submitted their work, unhide and show **slides 24–27**. Give students time to reflect on their thinking. Use student responses to see what misconceptions still exist.

Resources

- K20 Center. (n.d.). Bell Ringers and Exit Tickets. Strategies. https://learn.k20center.ou.edu/strategy/125
- K20 Center. (n.d.). Desmos Studio. Tech tools. https://learn.k20center.ou.edu/tech-tool/2356
- K20 Center. (n.d.). CPM Probability Generators. Tech Tools. https://learn.k20center.ou.edu/tech-tool/2317
- Vallinder, M. (2010, April 23). Simulation of a Zombie Outbreak! [Video]. YouTube. https://youtu.be/t_AlsNx5UhA