K20 LEARN

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# A Geometer's Perspective

## Trigonometric Ratios

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| **Grade Level** | 10th Grade | **Time Frame** | 2-3 class period(s) |
| **Subject** | Mathematics | **Duration** | 90 minutes |
| **Course** | Geometry, Precalculus | | |

## Essential Question

How are the angles and sides of a right triangle related?

## Summary

Students will investigate and discover the trigonometric ratios through observations about right triangles.

## Snapshot

### Engage

Students complete a card sort using triangle vocabulary.

### Explore

After seeing the need for another way to solve right triangles, students investigate the ratios of side lengths in similar right triangles.

### Explain

Students define sine, cosine, and tangent based on the relationships discovered in the exploration.

### Extend

Students use a clinometer to investigate angles in inclination and declination.

### Evaluate

Students demonstrate their knowledge of trigonometric ratios.

## Standards

*Oklahoma Academic Standards Mathematics (Geometry)*

**G.RT.1.2:** Verify and apply properties of right triangles, including properties of 45-45-90 and 30-60-90 triangles, to solve problems using algebraic and logical reasoning.
**G.RT.1.3:** Use the definition of the trigonometric functions to determine the sine, cosine, and tangent ratio of an acute angle in a right triangle. Apply the inverse trigonometric functions to find the measure of an acute angle in right triangles.
**G.RT.1.4:** Apply the trigonometric functions as ratios (sine, cosine, tangent) to find side lengths in right triangles in mathematical models, including the coordinate plane.

## Attachments

- Lesson Slides—A Geometers Perspective.pptx
- Right Triangle Relationships Observation Sheet Trigonometric Ratios—A Geometers Perspective - Spanish.docx
- Right Triangle Relationships Observation Sheet Trigonometric Ratios—A Geometers Perspective - Spanish.pdf
- Right Triangle Relationships Observation Sheet Trigonometric Ratios—A Geometers Perspective.docx
- Right Triangle Relationships Observation Sheet Trigonometric Ratios—A Geometers Perspective.pdf
- Sine Cosine Tangent Observation Sheet Trigonometric Ratios—A Geometers Perspective - Spanish.docx
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- Sine Cosine Tangent Observation Sheet Trigonometric Ratios—A Geometers Perspective.docx
- Sine Cosine Tangent Observation Sheet Trigonometric Ratios—A Geometers Perspective.pdf
- Triangle Card Sort Trigonometric Ratios—A Geometers Perspective - Spanish.docx
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- Triangle Card Sort Trigonometric Ratios—A Geometers Perspective.pdf
- Triangle Examples Trigonometric Ratios—A Geometers Perspective - Spanish.docx
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- Triangle Examples Trigonometric Ratios—A Geometers Perspective.pdf

## Materials

- Rulers
- Protractors or similar
- Scientific calculator
- Right Triangle Relationships Observation Sheet handout (attached; one per student)
- Sine Cosine Tangent Observation Sheet handout (attached; one per student)
- Triangle Card Spot handout (attached; one per group)
- Triangle Examples handout (attached; one per student)
- Clinometers

10 minutes

# Engage

Use **slides 3-4** to introduce the essential questions and lesson objectives as you see fit. Display **slide 5**. Form students into groups of 2-3 and provide them with a set of the **Triangle Card Sort**.

**Teacher's Note**

Ideally, students should not be aware of the mnemonic device SOHCAHTOA before this lesson.

**Teacher's Note**

A Card Sort is a formative assessment strategy from Keeley, 2008.

Allow students 5-15 minutes to sort the cards in their groups, allowing them to sort them in any manner that they find reasonable. You should not help students sort them, but instead question students about the sorting scheme they used and have them justify it. Choose a few groups to explain/share their sorting with the class.

**Teacher's Note**

Ideally, groups with different sorting schemes would share in order to reinforce that sound reasoning is the goal, not one particular "right answer."

If necessary, discuss other sorting schemes that become obvious after students have shared.

30 minutes

# Explore

Display **slide 6**. Display for the students a right triangle that is easily solved using the Pythagorean Theorem, and have the students walk through finding the solution. Make sure to have them explicitly state the information they're using and how they're using it.

### Teacher's Note

This may go something like, "We know how to solve this, and you've done one like this today in the Card Sort. How do we find the missing measure?" (An example has been provided under Attachments)

Display **slide 7**. After this discussion, display a right triangle that cannot be solved using only the Pythagorean Theorem. Pose the question to the class: "How can we solve this one?" The students will be puzzled, but they may have some idea or agree that they need more information. It cannot be solved with what is given. (An example has been provided under Attachments)

Display **slide 8**. Provide each student the attached **Right Triangle Relationships Observation Sheet**. Explain that by investigating right triangles, the class can figure out a way to solve the mysterious triangle. Allow students to work in their groups on this investigation. Provide help if needed, but try not to guide students explicitly. Instead, question the students about their process and thinking to help them come to their own conclusions.

### Teacher's Note

Some good questions to ask might be: Why do you think that? What does your group think? Can you tell me what you've tried already? What do you think you should do/try next? Tell me how the result was found? Do you think it holds if _____? What did you notice? What did you wonder?

It may be prudent to pause here and wait to begin the next portion during the next class period, as students will likely need more than 30 minutes to complete the investigation.

30 minutes

# Explain

Display **slide 9**. Bring the class back together for a discussion.

Display **slide 10**. Return to the "unsolvable" triangle and have students puzzle out how to solve it as a class, now that they have trigonometric ratios in their toolkit.

Have several students share their hypotheses about the ratio relationships they observed during their investigation.

Then, carefully facilitate a discussion about these relationships (it might be helpful first to discuss the origin of error(s) in their measurements) by asking students to demonstrate and explain how they formed their conclusions and how they tested their hypothesis using 30-60-90 triangles.

**Teacher's Note**

Example: "So, if we have an angle of 21 degrees, do we see that these ratios are the same, no matter the size of the triangle?" Draw one of the triangles from the investigation on the board/for students to see.

Ask the students leading questions: "Could your new understanding of right triangles help you solve for a missing side?" Have the students show you how. "Can we do this for any triangle with a 21 degree angle?" Yes, of course. "And then you figured out that it also works if your angle is 30 degrees or 60 degrees or 45 degrees?" Yes, they should have.

Find the missing side length in another example for a 30-60-90 or 45-45-90 triangle that the students used in their investigation using the ratios they agreed upon, or have a student draw and work it out while the class guides them.

Draw a triangle on the board with another angle measure (25 degrees in this example) and ask them to help with the equation: "Now that we know we can find the missing sides for a right triangle with a measure of 21 degrees, and even one with 30 degrees, what if the measure was 25 degrees? What would we have to do?"

**Teacher's Note**

Basically, we're asking the very important question: Do we have to do this ratio thing EVERY TIME?

Ask the students if there could be a shortcut and if so, what it would look like?

**Teacher's Note**

Hopefully they talk about functions, or equations, or multiplication instead of repeated addition. If not, guide them gently toward that idea, and then give them a minute to stew over this. They will most likely not have the answer yet.

Inform the students there is a shortcut, that there are established functions to show us the relationships between angles and side measures in a right triangle. These are called sine, cosine and tangent. Provide students the attached **Sine, Cosine, and Tangent Observation Sheet** and allow them to work it out. After students have determined the correct ratios, have them share with the class.

### Teacher's Note

Because the order for the ratios was not clearly defined in the first activity, be aware that students may have them "upside down." If students are not getting "matches" (or near matches) in the table on the "Sine, Cosine, and Tangent" observation sheet to the ratios they calculated on the first sheet, ask them what they might do differently to get a different value.

Return to the "unsolvable" triangle and have students puzzle out how to solve it as a class, now that they have trigonometric ratios in their toolkit.

10 minutes

# Extend

Display **slide 11**. Now with the understanding of trigonometric functions, students should create their own real world trigonometric function scenario for their peers to solve. They are not solving the problem, but creating it. On the bottom of the scenario, they need to write their definitions of sine, cosine, and tangent that they originally produced earlier in the lesson.

10 minutes

# Evaluate

Display **slide 12**. Students just completed making their trigonometric function word problem. Now, have students find a partner and solve their real world scenario using the knowledge they gained previously in the lesson on a separate sheet of paper. They should also analyze their peer's definition of sine, cosine, and tangent. They should check if their definitions are similar to theirs and determine how can they combine the two definitions to make one concrete definition for future reference. If they do not believe their peer's definition is accurate, they can examine that as well while creating a final draft of the definitions.

**Teacher's Note**

For example, some groups may use terminology like "short" leg or "long" leg that won't work out for large angles. Having students compare the definitions and think about which ones 'always' work, even if you move a triangle or have a very large reference angle, might be helpful.

## Resources


- "Finding Trigonometric Ratios." CPALMS lesson title used for inspiration. (2013-2015). Florida State University. http://www.cpalms.org/Public/PreviewResource/Preview/46546
- K20 Center. (n.d.). Card Sort. Strategies. https://learn.k20center.ou.edu/strategy/147