A Geometer's Perspective

Trigonometric Ratios

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Grade Level	9th – 10th Grade	**Time Frame**	3 class periods
Subject	Mathematics	**Duration**	125–135 minutes
Course	Geometry		

Essential Question

How are the angles and sides of a right triangle related?

Summary

Students will investigate and discover the trigonometric ratios (sine, cosine, and tangent) through observations of right triangles. Students are expected to know the Pythagorean theorem and its converse before beginning this lesson. This lesson is centered around the question: "How can we find missing measures in a right triangle if we cannot use the Pythagorean theorem?"

Snapshot

Engage

Students complete a card sort recalling triangle vocabulary.

Explore

After seeing the need for a way to solve right triangles—other than the Pythagorean theorem—students investigate the ratios of side lengths in similar right triangles.

Explain

Students define sine, cosine, and tangent based on the relationships discovered in the exploration.

Extend

Students apply their knowledge to solve problems involving trigonometric ratios.

Evaluate

Students demonstrate their knowledge of trigonometric ratios by solving for a missing side length.

Standards

ACT College and Career Readiness Standards - Mathematics (6-12)

G509: Express the sine, cosine, and tangent of an angle in a right triangle as a ratio of given side lengths

Oklahoma Academic Standards Mathematics (Geometry)

G.RT.1.3: Use the definition of the trigonometric functions to determine the sine, cosine, and tangent ratio of an acute angle in a right triangle. Apply the inverse trigonometric functions to find the measure of an acute angle in right triangles.

Attachments

- Lesson Slides—A Geometers Perspective.pptx
- Making Connections (Sample Responses)—A Geometers Perspective.docx
- Making Connections (Sample Responses)—A Geometers Perspective.pdf
- Making Connections—A Geometers Perspective - Spanish.docx
- Making Connections—A Geometers Perspective - Spanish.pdf
- Making Connections—A Geometers Perspective.docx
- Making Connections—A Geometers Perspective.pdf
- Right Triangle Exploration (Sample Responses)—A Geometers Perspective.docx
- Right Triangle Exploration (Sample Responses)—A Geometers Perspective.pdf
- Right Triangle Exploration—A Geometers Perspective - Spanish.docx
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- Right Triangle Exploration—A Geometers Perspective.pdf
- Triangle Cards—A Geometer's Perspective - Spanish.docx
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- Triangle Cards—A Geometer's Perspective.pdf
- Using Trig Ratios (Sample Responses)—A Geometer's Perspective.docx
- Using Trig Ratios (Sample Responses)—A Geometer's Perspective.pdf
- Using Trig Ratios—A Geometer's Perspective - Spanish.docx
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- Using Trig Ratios—A Geometer's Perspective.docx
- Using Trig Ratios—A Geometer's Perspective.pdf

Materials

- Lesson Slides (attached)
- Triangle Cards (attached; one per group; print one-sided)
- Right Triangle Exploration handout (attached; one per student; print one-sided)
- Right Triangle Exploration (Sample Responses) document (attached)
- Making Connections handout (attached; one per student; print one-sided)
- Making Connections (Sample Responses) document (attached)
- Using Trig Ratios handout (attached; one per student; printed one-sided)
- Using Trig Ratios (Sample Responses) document (attached)
- Pencils
- Paper
- Ruler (one per student)
- Scientific calculator (one per student)

Preparation

Before you begin, print the attached **Triangle Cards** (one copy per group of 2–3 students). Consider printing on cardstock paper, especially if you plan to reuse these cards. Cut out the cards; all of these cards are the same size for easy cutting.

15 minutes

Engage

Introduce the lesson using the attached **Lesson Slides**. Display **slide 3** to show the lesson's essential question. **Slide 4** identifies the lesson's learning objectives. Review each of these with your class to the extent you feel necessary.

Display **slide 5** and have students get a scientific calculator; follow regular classroom procedures for this. Form students into groups of 2-3 and provide them with a set of the attached **Triangle Cards**.

Introduce the Card Sort strategy to the class. Allow groups 5–10 minutes to sort the cards. Encourage them to sort the cards in any manner that they find reasonable. Advise students to use scratch paper to assist in this task. You should not help students sort the cards, but instead, question students about the sorting scheme they used and have them justify it. Choose a few groups to explain/share their sorting with the class.

Teacher's Note: Purpose

Ideally, groups with different sorting schemes will share in order to reinforce that sound reasoning is the goal, not one particular "right answer."

If necessary, discuss other sorting schemes that become obvious after students have shared.

30 minutes

Explore

Display **slide 6**, which has a right triangle that is easily solved using the Pythagorean theorem and knowledge of the sum of the interior angles of a triangle being 180°. Ask guiding questions and have volunteers help work through this problem as a class. Be sure to have students use academic vocabulary and explain not just how to do the next step, but also why.

Teacher's Note: Guiding the Activity

This may go something like, "We know how to solve this, and you have done one like this today in the Card Sort. How do we find the missing measure?"

After this discussion, display **slide 7**, which has a right triangle that cannot be solved using the Pythagorean theorem. Pose the question to the class: "How can we solve this one?" The students will be puzzled, but they may have some idea or agree that they need more information. Because it cannot be solved with their current knowledge, let's refer to this as the "unsolvable" triangle that we will return to later in the lesson.

Provide each student with a copy of the attached **Right Triangle Exploration** handout and a ruler. Explain that by investigating right triangles, the class can figure out a way to solve the mysterious "unsolvable" triangle. Allow students to work in their groups on this investigation. Transition through **slides 8–9** to review the directions for how they should gather the measurements to record in their table for each given triangle.

Teacher's Note: Purpose

Here, students are asked to use their ruler and prior knowledge to identify, measure, or calculate the values for the interior angles and side lengths of the given triangles. Students will use these values to find ratios and patterns to make a prediction. Use the attached **Right Triangle Exploration (Sample Responses)** document as needed.

Provide help if needed but try not to guide students explicitly. Instead, question the students about their process and help them come to their own conclusions. Consider asking some of the following questions:

- Why do you think that?
- What does your group think?
- Can you tell me what you've tried already?
- What do you think you should do/try next?
- How did you find the result?
- Do you think it holds if ___?
- What did you notice?
- What did you wonder?

As students finish gathering measurements, move to **slide 10** and direct their attention to the second page of their handout: "Comparing Data." Direct students to use their table from the first page and a calculator in order to write each ratio of segment lengths as a decimal.

After a few minutes, transition to **slide 11** and have students discuss their observations from the table. Then have them complete the handout by making a prediction about the relationship between the ratios and the sides of the right triangles. As time allows, have students share their predictions with the class.

Teacher's Note: Pacing

If you have a traditional 45-minute class period, it may be prudent to pause here and wait to begin the next portion during the next class period, as students might need more than 30 minutes to complete the exploration activity.

Have students keep their handout in a safe place as they need it for the Explain portion of the lesson.

40 minutes

Explain

Teacher's Note: Guiding the Lesson

For the following activity, it is best if students continue working with the same small group of 2-3 that they did during the Explore portion of this lesson. Students will also need to still have access to a scientific calculator, but they will no longer need a ruler.

Give each student a copy of the attached **Making Connections** handout. Transition through **slides 12–13** to preview the activity with the class. They need to use the first table from the Right Triangle Exploration handout from the Explore portion of the lesson to complete the first column of their table on their Making Connections handout. They are then to use their calculator to complete the remainder of the table with the goal of writing their own definitions for sine, cosine, and tangent.

Display **slide 14** and have students find the buttons on their calculator needed to find the sine, cosine, and tangent ratios of given angles. Direct students' attention to the first row of the table. Tell the class that for some triangle *XYZ*, they are given that angle *Y* is 33°. Have students use their calculator to find the sine, cosine, and tangent of 33°. Tell them to check their work with that first row. In other words, if they find that sin (33°) = 0.54, then they are using their calculator correctly, which is the purpose of this first row of the table. Once students understand how to use their calculator, have them complete the table. As students are working, circulate the room and answer any questions about how to use the calculator, but wait to answer other questions, as students will be noticing patterns in this activity as well.

As students are completing the table, transition to **slide 15**. Have groups compare their tables with the ones from the second page of their Right Triangle Exploration handout. Direct groups to discuss the similarities between the tables and jot down a few notes on their handout.

Move to **slide 16** and introduce the vocabulary of *opposite*, *adjacent*, and *hypotenuse*, using the graphics on the slide. Ask for volunteers to share why they think the base of the two triangles are labeled differently. Facilitate a brief discussion but be sure that students understand that the angle is what determines which side is the opposite and which side is the adjacent side before continuing the lesson. Encourage students to take notes on the back of the Making Connections handout.

Show **slide 17** and have groups write their own definitions for *sine*, *cosine*, and *tangent* using their new vocabulary: *opposite*, *adjacent*, and *hypotenuse* on the back of their handouts.

Move to **slide 18** and have groups trade their handouts/definitions with another group. Then direct students to use the definition they were given—and only that definition—to find the sine, cosine, and tangent of angle *A* from the triangle on the slide. Have students find these ratios on a piece of notebook paper, rather than write on their peer's paper.

Show **slide 19** and have students trade handouts/definitions back and then compare their results. Ask the class if they got the same results or not, and if they did not, have groups share with each other how they might clarify their definition.

After students revise their definitions, use **slides 20–21** to instruct students to repeat this task with a different group.

Once students have their own handouts and have had time to again revise their definitions, transition through **slides 22–24** to give students the definitions for the ratios of sine, cosine, and tangent. Give students time to edit their definitions as needed.

Use the attached **Making Connections (Sample Responses)** document as needed.

Teacher's Note: Purpose

The purpose of this activity is for students to gain their own understanding of these trigonometric ratios. When students write definitions in their own words, it can significantly help with retention. It is also important for students to see why clarity is important in their definitions. For example, some groups may use terminology like "short" leg or "long" leg; help them see that those types of descriptions do not work for all triangles. Having students compare the definitions and think about which ones “always” work, even if you move a triangle or have a very large reference angle, will be helpful.

35 minutes

Extend

Place students into groups of four and then introduce the Numbered Heads Together strategy. Assign each student a number, 1–4, within each group. Display **slide 25** and give each student a copy of the **Using Trig Ratios** handout. Explain to the class that after each question, everyone in each group should be ready to share their group's answer to the problem and reasoning.

Explain to students that they have approximately three minutes to complete the first question. Remind the class that everyone needs to be ready to share. Then have everyone who was assigned number 2 take turns sharing how their group answered the first question. Repeat this with the remaining three questions on the handout, each time selecting a different assigned number.

Sample Student Responses:

- **Question 1:** We used the Pythagorean theorem to find the hypotenuse; then we found the cosine of *F* as the side adjacent to angle *F* over the hypotenuse.
- **Question 2:** The given ratio had the hypotenuse on the bottom of the fraction, so it had to involve sine or cosine. It could have been the sine of angle *C* or the cosine of angle *A*, so option (b) *sin*(*C*) was the right choice.
- **Question 3:** We had to draw a triangle and use the information about sine to label the side opposite of θ and the hypotenuse. We did the same thing for the adjacent side, using cosine. Then we found the *tan*(θ).
- **Question 4:** This one was hard. We found the sin(N) was 8 over the hypotenuse, but the problem said that the sin(N) was two-thirds. So that meant that 8 over something had to reduce to two-thirds, so we wrote a proportion to find the hypotenuse.

Use the **Using Trig Ratios (Sample Responses)** document as a guide if needed.

5 minutes

Evaluate

Ask the class: *Could your new understanding of right triangles help you solve for a missing side?* Display **slide 26** and return to the "unsolvable" triangle. Use the Exit Ticket strategy to individually assess what students have learned from the lesson. Have students independently find the missing side length, now that they have trigonometric ratios in their toolkit. Allow students a couple of minutes to respond using an index card, sticky note, piece of paper, etc.

Use the hidden **slide 27** to check students' work. Consider unhiding the slide and using it the following day as bellwork. Use students' responses as a formative assessment to see if they are ready for the next topic or need additional practice.

Optional Addition:

Consider giving students additional practice by having them use a clinometer to investigate angles of inclination or declination then use their knowledge of trigonometric ratios to calculate distances from objects or heights of objects. Alternatively, students could create their own real-world trigonometric scenario for their peers to solve. Then they could trade papers and solve each other's created problems.

Resources

- K20 Center. (n.d.). Bell ringers and exit tickets. Strategies. https://learn.k20center.ou.edu/strategy/125
- K20 Center. (n.d.). Card sort. Strategies. https://learn.k20center.ou.edu/strategy/147
- K20 Center. (n.d.). Numbered heads together. Strategies. https://learn.k20center.ou.edu/strategy/2476
- Wallace, K., Meyers, C. (2015). *Discovering trigonometric ratios*. [Lesson]. Florida State University. http://www.cpalms.org/Public/PreviewResource/Preview/46546