## Trashketball: Part 1

 Scatter Plots and Lines of Best Fit

K20 Center, Kate Raymond, Levi Patrick, Samantha Marshall<br>Published by K20 Center

This work is licensed under a Creative Commons CC BY-SA 4.0 License

| Grade Level | 8th -9 th Grade | Time Frame | 200 minutes |
| :--- | :--- | :--- | :--- |
| Subject | Mathematics | Duration | 3-5 class periods |
| Course | Algebra 1, Middle School Mathematics |  |  |

## Essential Question

How can linear relations and data analysis help us to understand the abilities of people?

## Summary

In this lesson, students toss balls of trash, or "trashballs," into the trashcan, recording their shooting percentage at various distances from the basket. With distances measured and percentages figured, students create a scatter plot and a line of best fit to make a linear model of the shooting skills of the class. This is a great, simple, and inexpensive lesson that addresses many significant algebra 1 concepts, including expressions, data analysis, scatter plots, and linear equations. Students should understand how to write the equation of a line, given its graph, before participating in this lesson.

## Snapshot

## Engage

Students analyze data that can be used to maximize scores in bowling.

## Explore

Students create an algebraic expression for shooting percentages and make a prediction about shooting percentage. They use their expression to verify their prediction and make more predictions about the relationship between the distance from the basket and scoring percentage. Finally, students conduct an experiment and gather data to test their hypothesis.

## Explain

Students use their formula and the data they collected to create a scatter plot for the relationship between shooting percentage and distance from the basket.

## Extend

Students work together to define correlation, positive correlation, negative correlation, and relatively no correlation. Students use these terms to describe the relationship illustrated by their scatter plots. They then work to create a definition of the term "line of best fit," draw a hypothesis for the line of best fit for the data, and write the equation for their line of best fit. Students then use technology to find the line of best fit.

## Evaluate

Students compare and contrast the line of best fit they found by hand with the line of best fit they found using technology. They then use the line of best fit to make predictions about shooting percentages.

## Standards

Oklahoma Academic Standards for Mathematics (Grade 8)
PA.A.1.1: Recognize that a function is a relationship between an independent variable and a dependent variable in which the value of the independent variable determines the value of the dependent variable.
PA.A.1.2: Use linear functions to represent and explain real-world and mathematical situations.
Oklahoma Academic Standards for Mathematics (Grade 8)
A1.F.1.2: Identify the dependent and independent variables as well as the domain and range given a function, equation, or graph. Identify restrictions on the domain and range in real-world contexts. A1.D.1.2: Collect data and use scatterplots to analyze patterns and describe linear relationships between two variables. Using graphing technology, determine regression lines and correlation coefficients; use regression lines to make predictions and correlation coefficients to assess the reliability of those predictions.

## Attachments

- Bowling with Jacob - Spanish.docx
- Bowling with Jacob - Spanish.pdf
- Bowling with Jacob.docx
- Bowling with Jacob.pdf
- Trashketball Handout - Spanish.docx
- Trashketball Handout - Spanish.pdf
- Trashketball Handout.docx
- Trashketball Handout.pdf
- Using Technology to Create a Line of Best Fit - Spanish.docx
- Using Technology to Create a Line of Best Fit - Spanish.pdf
- Using Technology to Create a Line of Best Fit.docx
- Using Technology to Create a Line of Best Fit.pdf
- Vocabulary for Teachers.docx
- Vocabulary for Teachers.pdf


## Materials

- Blank paper (one for each pair of students)
- Green sticky notes (one per student)
- Red sticky notes (one per student)
- Wastepaper basket
- Tape measures (at least one for every three students; ideally one per student)
- Graphing calculators or spreadsheet software
- Bowling With Jacob handout (attached)
- Trashketball handout (attached)


## Engage

Divide students into groups of two or three. Pass out the attached Bowling With Jacob handout. Have students work in their groups to complete the handout. Each student should record their responses on the handout.

Ask each group to discuss their response to the first question. After all groups have given their answers and reasoning, ask students if they noted any differences in the responses. Discuss any differences noted.

Repeat the above process for the other two questions on the handout.

## Teacher's Note

The point of this exercise is to engage students and to assess their prior knowledge about scatter plots and the use of data.

## Explore

Inform students they will be investigating data about shooting percentages. Ask students to work with a partner to create a definition for shooting percentage.

After a few minutes, ask for volunteers to share their definitions. Record the given definition on the board, and ask other pairs how it differs from their own. Ask if any pair thinks the definition needs to be altered. Continue the discussion until the class agrees on the definition.

Have students work with their partners to create a mathematical definition, or formula, for shooting percentage.

## Teacher's Note

As students work on creating a shooting percentage formula, circulate around the room, asking questions to clarify student thinking and reasoning. Common questions include: What information do we need in order to find a shooting percentage? Is that value a percentage? What can we do with these values in order to make the formula reflect the definition we created?

Have each of the student pairs write their formulas on a standard size piece of paper, large enough for the class to see. Post these papers around the room (or have students hold them up) so the other groups can view them. Ask if all of the formulas are equivalent or if there are differences. Have students discuss the differences they see. Ask students if these differences would result in different shooting percentage values and have them explain their thinking. Through this discussion, students should work together to create one formula for the class to use to determine shooting percentages.

Make sure students save the papers on which they wrote their formulas. They will use these papers later to play trashketball.

## Teacher's Note: Common Obstacles

Some students may write formulas that require a certain number of attempted shots (100 is most common). If so, compare the formula they created with one that takes into account different numbers of attempted shots. Ask if the two formulas would result in the same value if 100 shots were attempted, and then ask if you could still use both formulas if the shooter than took another 10 shots, for a total of 110 attempts. Students should reach the conclusion that allowing the shooter to attempt different numbers of shots makes the formula more flexible and useful. Other students may have written formulas that express a shooting ratio but not a percentage. You can have students do an example problem and check how reasonable their answer is to help them identify this problem. For example, if students write that a shooting percentage is solely determined by shots made divided by shots attempted, ask them what the shooting percentage would be if someone made 8 out of 10 shots. Their formula will result in 0.8 . Point out that $0.8 \%$ is a very low percentage, especially considering the shooter made the majority of these attempts.

Give each student a green sticky note and a red sticky note. For each of the examples below, have students hold up a green sticky if they think the example should result in a high shooting percentage, or a red sticky if they think the example should result in a low shooting percentage.

## No Sticky Notes? No Problem!

If no sticky notes are available, you can simply call out "high" and "low" and have students raise their hands when they agree.

Example 1: 7 attempts, 3 shots made
Example 2: 9 attempts, 8 shots made
Example 3: 5 attempts, 0 shots made

## Teacher's Note

Pause here to ask if anyone can predict the exact value of the shooting percentage for the above example. Students should recognize that the shooting percentage is $0 \%$, since zero shots were made.

Example 4: 12 attempts, 12 shots made

## Teacher's Note

Pause here and again ask if anyone can predict the exact value of the shooting percentage for this example. Students should recognize that the shooting percentage is $100 \%$, since every attempted shot was made.

Example 5: 15 attempts, 16 shots made

## Teacher's Note

Pause here and ask if this data seems reasonable. It should not be possible to make more shots than you attempted. Ask students if they can make a prediction about what the shooting percentage will be if they calculate it with this data. Students should recognize that it would turn out to be more than $100 \%$.

Once they have made a prediction for each example, have students use their formula to calculate the shooting percentage. Have students verify that the values they get from the formula match the predictions they made.

Once the class is convinced that the formula they created accurately measures shooting percentage, introduce students to the game of trashketball. Place an empty wastebasket somewhere in the room so that there is space free of obstacles in front of it. The wastebasket can be placed on a desk, table, or on the floor. Wad up a piece of paper and demonstrate shooting it into the wastebasket and retrieving it afterwards.

Ask students to consider if and how moving away from the basket will affect the shooting percentage. Tell students to use the words increase, decrease, or constant to create a hypothesis that describes what will happen to the shooting percentage as the distance between the shooter and the basket increases. Students should record their hypotheses as a complete sentence on the attached Trashketball handout.

## Sample Hypotheses

As the distance to the basket increases, the shooting percentage will also increase. As the distance between the basket and the shooter increases, the shooting percentage will remain the same. As the distance between the basket and the shooter increases, the shooting percentage will decrease.

Ask students to identify the independent and dependent variables for their hypotheses and record them on their handouts for question 2.

## Teacher's Note

It helps to emphasize that that dependent variable depends on the independent variable. You can also point out that the independent variable tends to be found in the premise of the hypothesis (first clause) and the dependent variable is usually found in the conclusion of the hypothesis (second clause).

Ask students to identify the control variables and record these for question 2.

## Teacher's Note

Emphasize that control variables could affect the results of the experiment, unless they are controlled and forced to remain constant. Formal definition of all of these terms can be found on the "Vocabulary for Teachers" resource (located under Attachments).

Move students, an empty wastebasket, and several tape measures outside (or to the gym, cafeteria, or other area of the school with a large floor space). Have students bring their Trashketball handouts, a writing utensil, and the papers on which they wrote their shooting percentage formulas.

Place the wastebasket in the middle of the floor and use the tape measure to measure a radius of 8 feet from the basket. Have students form a circle around the basket at that distance, standing next to the partner with whom they originally wrote their shooting percentage formulas.

Tell students to tear their formula papers in two. Each partner in a pair gets half of their paper to make a trashketball. Explain that every student will crumple up the half-sheet of paper and try to shoot it into the wastebasket. Ask students what that means about the values in the chart for question 3 of their Trashketball handout. Students should be able to explain that the attempts made will equal the number of students in the class. Have them record this information on their charts and then ask students to make their first attempt from 8 feet.

## Teacher's Note

You may choose to have all students shoot simultaneously to decrease student anxiety about making or missing it, since they won't be observing each other as they shoot. However, this sometimes causes issues-especially when using a smaller wastepaper basket—as papers will collide with each other. Another option is to have students count off, and then have them shoot when their number is called. Calling off numbers in rapid succession allows students to shoot quickly (and avoid being watched) but at different times.

Once all students have taken their shots, go to the wastebasket and count out how many wads of paper made it into the wastebasket. Have students retrieve a wad of paper (it does not have to be the one they threw) and then form a new circle nine feet from the basket. Continue shooting, recording, and forming circles at distances increasing by 1 foot each time until you run out of room in your space or on the chart.

## Teacher's Note

You can have students shoot from distances smaller than 8 feet as well, but this might mean splitting the class into two groups and having the groups take turns, as there will be less room in circles formed at smaller distances. A good rule of thumb is to multiply the radius of the circle by four to determine how many students can comfortably fit around it.

Once all of the data is collected, let students work in groups of two or three to find the shooting percentages for question 3 of the handout. Students may use calculators but should be sure to check their group's work before recording it on their own papers.

Once all of the shooting percentages are calculated, call on one group at a time to share their result for the shooting percentage at a specific distance. Have the other groups check the other groups' work. All groups should have the same result. If not, have disagreeing groups share their work and reasoning to settle differences. Continue calling on groups to share results until the class agrees on all of the shooting percentages. Once the class has reached agreement, have each student use the data to create a scatter plot for question 4 on the Trashketball handout.

## Explain

Once all students have created their scatter plots, use the I Notice, I Wonder strategy to elicit their thinking.

## Sample Student Notices and Wonderings

Students should notice a relationship between distance from the basket and shooting percentages. They may say something like, "We make fewer shots as we get farther away." You may want to rephrase this so that proper mathematics terminology is used. For example, you could say "So, what you are saying is that as our distance from the basket increases, our shooting percentage decreases." Students may wonder what would happen if they were able to shoot from a closer range or from farther away. They may ask how far away they would need to get before the shooting percentage is 0 or how close they would need to be for the shooting percentage to be 100. Record all of these wonderings somewhere students can see them, but they do not need to be addressed at this time.

Explain to students that what they have noticed about the data is that there is a correlation, or relationship, between the variables. Ask students to work in pairs or groups of three to create a definition of correlation.

After every group has created a definition, have one member from each group share their group's definition.

After all groups have shared, ask students what most (or all) definitions had in common, what seems necessary to include in the definition, and what is unnecessary. Create one class definition based on this discussion.

Read the formal definition of correlation given in the "Vocabulary for Teachers" attachment for this lesson. Ask students to compare and contrast their definitions with the formal definition. Allow students to edit the class definition as needed. Once everyone is satisfied with the class definition, have students record it on their Trashketball handouts.

Ask students to return to their groups and consider the next three terms on the handout. Have each group create their own definitions for each term and repeat the procedure above to create class definitions of these three terms. Have students record these definitions on their handouts.

## Teacher's Note

Encourage students to use the terms "independent variable" and "dependent variable" in their definitions. It is important for students to understand that a positive correlation means that the dependent variable increases as the independent variable increases and a negative correlation means that the dependent variable decreases as the independent variable increases. Another way students might express this is that the independent and dependent variables exhibit the same behavior for positive correlations and opposite behaviors for negative correlations. Expressing the relations this way may save students confusion if they accidentally travel from right to left along a relation, rather than left to right.

Have students work in pairs to answer questions 5 and 6 on their handouts.
Ask students to raise their hands if they found relatively no correlation in the data. If any groups raise their hand, ask one member of those groups to explain their group's reasoning to the class. Allow other students to ask that student questions or to analyze the group's reasoning.

## Teacher's Note

It may be helpful to allow students to use a document camera to display their handouts as they explain their reasoning to the class.

Next, ask student to raise their hands if they found that there was a positive correlation in the data. If any groups raise their hand, ask one member of those groups to explain their group's reasoning to the class. Allow other students to ask that student questions or to analyze the group's reasoning.

Finally, ask students to raise their hands if they found that there was a negative correlation in the data. If any groups raise their hand, ask one member of those groups to explain their group's reasoning to the class. Allow other students to ask that student questions or to analyze the group's reasoning.

After all three views have been presented and analyzed, poll the class to determine if everyone agrees on the kind of correlation the data represents. If not, continue the discussion until a consensus is reached.

## Teacher's Note

Students should reach the conclusion that shooting percentage and distance from the basket are negatively correlated.

## Extend

Point out to students that their data is close to being linear, but it is not perfectly linear. Tell students that, in cases like this, they will need to work with a line of best fit. Ask students to work in pairs or groups of three to consider the term "line of best fit." What could the definition be? Why would a line of best fit be useful? What would some of the characteristics of a line of best fit be?

After several minutes, each group should share their definitions of a line of best fit. Discuss each definition and what they have in common. Use this discussion to create a class definition of a line of best fit. Have students record this definition on question 7 of the handout.

Next, have each group share one characteristic of a line of best fit. Record the characteristics somewhere all students can see.

## Possible Characteristics Observed by Students

The slope of a line of best fit should be positive for positive correlations and negative for negative correlations. The line should pass through as many points as possible. The line should have the same number of points above and below it.

Ask students to pick the 3 most important characteristics listed on the board. Discuss these characteristics until the class comes to an agreement about which three are most important. Have students record these characteristics on question 8 of the handout.

Then, direct students to complete question 9 on the handout by drawing a line that fits as many of the listed characteristics as possible. Finally, have students complete question 10 on the handout by writing the equation of the line they drew.

In pairs or groups of three, have students meet in groups to discuss their lines of best fit and complete question 11 on their handouts.

Discuss with the class the similarities and differences they found. Ask students if they can be sure of which line best fits the data. Students should come to realize that, although their equations are all similar, they do not yet have the ability to choose a "best" line.

Explain to students that computer programs and calculators have the ability to find a "best" fit by examining every possible line and finding the one line that minimizes the distance between it and the points on the scatterplot. Demonstrate how to find a line of best fit (linear regression) using a graphing calculator or spreadsheet software and the data from the Bowling With Jacob handout.

## Teacher's Note

The document "Using technology to Create a Line of Best Fit" (located under Attachments) gives step-by-step directions for using Microsoft Excel or a TI-84 and is attached to this lesson. Video explanations can also be found in the resource section.

After demonstrating using the data from the Bowling With Jacob handout, have students find the line of best fit for the trashketball data.

## Evaluate

Call on one student to share the equation they found for the line of best fit using technology. Have other students verify that they found the same equation and the same line of best fit. If there is disagreement, have students show how they got a different equation. This may involve having students read off the coordinates they used as input, as this is usually the reason for discrepancies.

Have all students use a red pen or marker to graph the new line of best fit found using technology on their scatter plots. Ask students how graphs of the two lines of best fit compare. Which one do they think is a better line of best fit and why?

In groups of two or three, have students complete questions 12 through 14 on their handouts. After all groups complete these questions, have groups share their responses and discuss them with the class.

Allow the groups to complete question 15. After all groups complete these questions, have groups share their responses and discuss them with the class.

## Resources

- K20 Center. (n.d.). I Notice, I Wonder. Strategies. https://learn.k20center.ou.edu/strategy/180
- Video showing how to create a line of best fit in Microsoft Excel: Computingboss. (2013, March 31). Microsoft Excel 2010 - Line of best fit \& equation [Video]. YouTube. https://www.youtube.com/watch? v=Ogx7C 11 JD9k
- Video showing how to create a line of best fit on a TI-84: Graphing Calculator Review. (2012, March 29). Find line of best fit on a TI-84: How to guide [Video]. YouTube. https://www.youtube.com/watch? $\mathrm{v}=$ HTFtogVoLiw

