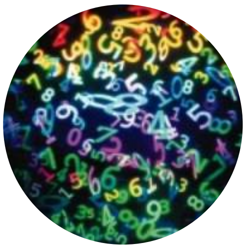
K20 L•E•A•R•N



Prime Time

Prime Factorization

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Grade Level	6th Grade	**Time Frame**	4-5 class period(s)
Subject	Mathematics	**Duration**	235 minutes
Course	Middle School Mathematics		

Essential Question

What can arrays tell us about the nature of numbers?

Summary

This lesson builds a conceptual understanding of prime and composite numbers through arrays, leading to a clearer understanding of greatest common factors, least common multiples, and reduced fractions.

Snapshot

Engage

Students are given a situation in which an array must be created and generate observations and questions about how the array could be created.

Explore

Students form arrays to represent a particular number and explain how they know they have found all of the arrays. Students look at the arrays formed for various numbers and attempt to find patterns and relationships.

Explain

Students compare prime factors and composite factors to determine and explain the uniqueness of a prime factorization.

Extend

Students work to develop their own methods of finding prime factorization by working in groups and comparing the methods used by each group.

Evaluate

Students find the prime factorization of numbers 1-100 and compare the factorizations on a number chart.

Standards

Oklahoma Academic Standards for Computer Science (Fifth Grade)

5.AP.V: Variables

Oklahoma Academic Standards for Mathematics (Grade 6)

6.N.1.5: Factor whole numbers and express prime and composite numbers as a product of prime factors with exponents.

Attachments

- Engage Slide.pdf
- Engage Slide.pptx
- Explore Number Cards - Spanish.docx
- Explore Number Cards - Spanish.pdf
- Explore Number Cards.docx
- Explore Number Cards.pdf
- Factor Comparison Chart - Spanish.docx
- Factor Comparison Chart - Spanish.pdf
- Factor Comparison Chart.docx
- Factor Comparison Chart.pdf

Materials

- Engage Slide
- Graph paper
- Colored pencils or markers
- Glue sticks
- Number cards from "Explore Number Cards" attachment
- Document camera
- "Factor Comparison Chart" attachment
- Sticky notes
- One large classroom number chart (1 to 100)

Engage

Show the attached **Engage slide** on a SMART board or projector screen and read it to your class. Use the I Notice, I Wonder strategy to engage students in thinking about the situation presented. After you record their notices, ask students to identify the things they noticed that are mathematical. Repeat this procedure with their wonderings.

Students are likely to point out the notices that include numbers (e.g., there must be the same number of chairs in each row) as being mathematical but are unlikely to point to notices or wonderings that refer to the actual arrangement of chairs. Point this out to students, and help them understand that ways of arranging chairs also depend on mathematics.

For each wondering listed, ask students what they would need to know in order to figure out the answer to their wondering. Students may not be able to answer these questions at this time, but it is important to ask the question to engage the students in thinking how they might go about answering the questions they posed.

Explore

Print and cut apart the number cards on the **Explore Number Cards** attachment. Group students into pairs or threes and randomly pass out one card to each group. Tell students that the number on the card is the number of students in the graduating class.

Pass out graph paper and colored pencils or markers. Have students paste down the number they were given onto the piece of graph paper. Having the numbers on the page will help in later parts of the lesson when students compare the arrays of more than one number.

Tell students they are to show all the different possible arrangements of chairs for their number of graduates by shading in a square for each chair.

Teacher's Note

You may want to remind students that this means each column and row must have an equal number of seats. You may also want to tell students not to "skip" squares between rows and columns, as they will quickly run out of space on their graph paper.

Allow students to work in groups until all groups believe they have found all of the configurations for their number of graduates.

Teacher's Note

If one or two groups finish very quickly and you haven't used all the number cards, you can assign them a second number with which to work. Be sure to give them new graph paper for this number. One or two groups that are given composite numbers with a large about of factors may need more than one piece of graph paper.

Have students present their results to the class one by one by placing their graph paper under the document camera. As they present, ask each group how they know there aren't any more ways to arrange the chairs. Be sure that they can explain how they know they have found all of the arrangements.

Teacher's Note

Pay careful attention to whether groups represent arrays with rows and column interchanged (for example, 4 x 6 and 6 x 4) as the same array or two different arrays. Make sure to question students about their thinking on this matter. Ideally, students should decide that the arrays are different, especially since they are basing this on a real-life situation in which orientation matters (since graduates will be facing a particular direction).

Optional

You may also want to ask each group which arrangement they think would look or work best. This is a good lead-in to the "rectangle pageant" in the lesson "Are We Golden?"

After each group presents, have them hang their graph paper in a designated area of the room.

After all the groups present, have students examine the collection of graph papers now hanging on the wall and try to find some similarities.

Explain

Have students explain the similarities they have found.

> **Teacher's Note**
>
> They should notice that all of the graph papers include an array where the number or rows is one and an array where the number of columns is one. Explain to students that the number of rows/columns that can be used to make an array for a given number is called a factor of that number and that one is a factor of every number.

Ask students to look at the numbers that paired with one in the arrays and tell you what they notice. They should notice that this number always is equal to the given number of graduates. Connect this idea with the fact that one is the multiplicative identity: one times any number gives that number.

Next, tell students that the class needs to rearrange the posters so that the arrangement makes sense. Do not be explicit about what may or may not make sense. Let students form conclusions about this on their own. Once the class has arranged the graph paper in a way that makes sense to them, discuss their arrangement with them and have them explain why the arrangement makes sense to them.

Hopefully, their arrangement indicates, in some way, that 53 is "different" than any others because it is the only one with just two possible arrangements of chairs: 1 x 53 and 53 x 1. Tell them that a number with only two possible arrangements is called prime and has only two factors: itself and one. Numbers for which more than two arrays can be made are called composite numbers.

Tell groups to send one person to retrieve the graph paper for their number(s). Have groups list all of the factors of their numbers in their notebooks.

> **Teacher's Note**
>
> They should list any number that is equal to the number of rows or columns in one of their arrays.

Ask students to compare the number of factors they found with the numbers of arrays they were able to make for their number. They should notice that these are the same—that is, there is exactly one array per factor.

Next, tell groups to use the back side of their piece of graph paper to determine which of these factors are prime (can only make two arrays) and which factors are composite (can make more than two arrays) .

Call on one group at a time to share which factors they believe are prime and which they believe are composite. Put the chart provided in **Factor Comparison Chart** under the document camera and record which numbers they say are prime and which numbers they say are composite. If a group says a number is prime that another group has already listed as composite (or vice versa), point out this issue. Ask the group that reported the number as a composite number to share the arrays they made. Check their work to confirm or disprove that the number is composite. Record the number in the appropriate space and continue calling on groups to share their results until all groups have reported.

After students have determined which factors of their numbers are prime and which are composite, tell students that their goal is to write their original number as a product of only the prime factors of that number. They may use each prime number more than once, if they wish, but cannot use a composite factor. Tell students this is called writing a prime factorization.

After each group has found their prime factorization, ask if there are any groups that did not use all of their prime factors at least once. All groups should respond that they used all of their factors.

Next, ask students to find other ways to multiply together some or all of their prime factors (using them more than once, if necessary) so that the product is their original number. Allow students some time to think and struggle with this.

Teacher's Note

Students will often ask if "order matters" for this exercise. In other words, is 2 x 3 a different factorization than 3 x 2? This is a great opportunity to discuss the commutative (turn around) property of multiplication with your students. Given that property, we should consider 2 x 3 and 3 x 2 as representing the same factorization.

Ultimately, students will become frustrated when they are unable to create a second prime factorization. Ask students if they think it is possible that a number has more than one prime factorization until they become convinced that there is only one prime factorization per number.

Ask students if they would be able to write more factorizations if they were to use composite factors as well as prime factors. Most students will readily agree that they could do that. Ask each group to come up with a factorization that uses at least one composite factor.

Teacher's Note

Several groups will be unable to do this. As they begin to realize they can't create a composite factorization, tell them to think about reasons why they can't make a composite factorization as the other groups create their factorizations.

After students have created composite factorizations, call on a group to share their composite factorization.

Write their composite factorization on the board. Then, ask them to share their prime factorization. Write their prime factorization under the composite factorization. Ask students to think about the way in which the two factorizations are related. As you discuss this question with them, they should come to realize that the prime numbers in the prime factorization are factors of the composite numbers in the composite factorization. Show the students how the composite factorization can be "broken down" into the prime factorization.

After having many groups share composite factorizations and break them down, call on a group that was not able to write a composite factorization. Have them explain why they were unable to do this. They should be able to explain that they had no composite factors to use in the first place.

Recap the following for students.

We found that:

- A prime factorization can be written for every number.
- No number has more than one prime factorization.
- Some numbers have no composite factorizations.
- Some numbers have more than one composite factorization.

Tell students that mathematicians summarize the information in the first two bullet points above by saying there is a unique prime factorization for every number.

Extend

Tell students that their next task it to figure out a method they can use to determine the unique prime factorization for any number. Tell students to start by working in pairs or threes to find the prime factorization of 72. As they do so, they should be careful to think about the method they are using as they will be asked to explain their methods to the rest of class.

Wait until most groups seem to have found the prime factorization. Call on groups one at a time to share their method and the prime factorization they found. Refrain from commenting on the methods or results, but encourage students to ask questions about the method or the results.

After all the groups have shared, ask the class if all of the prime factorizations were the same. They should be able to tell you that, aside from differences in order, they were the same. If there is disagreement about this point, discuss it until everyone is confident that the prime factorizations are all the same.

Teacher's Note

At this point, you might want to insist that prime factors are listed least to greatest, so future comparison can be easier.

Ask the class which methods they like or dislike and why. Discuss the various methods.

Next, ask the class to pick whatever method they best understood or most liked and use it to factor 96.

After several minutes, have groups discuss their results and methods, including any problems they encountered or changes they made to the method.

Repeat this process for the numbers 80, 75, and 98 or until all pairs seem to be using efficient and similar methods. This method should look much like the traditional "factor tree" method of finding prime factorization.

Tell students you are going to demonstrate one more method of prime factorization to them. Demonstrate the factor tree method (or your preferred method) using the numbers 48, 56, and 36.

Evaluate

Divide the cards numbered 1-100 evenly between students. For each number you give a student, give them a sticky note note too. Tell them to write their name on the back of each sticky note.

Have the students write the prime factorizations for each of the numbers they received on the front of the sticky note (one factorization per sticky note). As they complete each factorization, they should post their note in the corresponding space on the number chart.

After all the factorizations have been posted, have students look for patterns in the chart. Discuss any patterns they find.

Teacher's Note

There are many, many patterns to be found in a number chart. There are no "right" patterns or "wrong" patterns to find. The point is to engage students in searching for patterns. See the resources for some examples.

Resources

- Gaskins, S. (2008, September 22). 30+ things to do with a hundred chart [Web log]. Retrieved from http://letsplaymath.net/2008/09/22/things-to-do-hundred-chart/
- K20 Center. (n.d.). I Notice, I Wonder. https://learn.k20center.ou.edu/strategy/d9908066f654727934df7bf4f507d1a7
- Mattox, D., Biddy, Q., & Reeder, S. (n.d.). Are We Golden? K20 LEARN. https://learn.k20center.ou.edu/lesson/59169e1415dae6976ef2d0d6f102218c
- New Zealand Government. (n.d.). Factor patterns. nzmaths. http://nzmaths.co.nz/resource/factor-patterns
- Whealton, S. (n.d.). Prime factor configuration pattern numbers. http://www.washingtonart.net/whealton/PFCN.html