

Meet Me in the Middle

Discovering Midpoint Formula Through Estimations

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Grade Level	9th – 10th Grade	**Time Frame**	125 minutes
Subject	Mathematics	**Duration**	3 class periods
Course	Geometry, Intermediate Algebra		

Essential Question

How can we mathematically determine when something is "in the middle?"

Summary

In this lesson, students will estimate population data leading them to discover the midpoint formula. They will then apply the formula they created to various other real-world applications.

Snapshot

Engage

Students estimate the population of their school, town, state, country, and world.

Explore

Students use census data to estimate populations for several years and, in doing so, begin to construct a formula for a midpoint.

Explain

Students explain their method for estimation and discuss the different approaches used by their classmates to refine their definition and formula for midpoint.

Extend

Students brainstorm about the usefulness of a midpoint formula and apply their constructed formula to several real-life applications

Evaluate

Students use their midpoint formula to justify a decision about a road trip.

Standards

ACT College and Career Readiness Standards - Mathematics (6-12)

G511: Find the midpoint of a line segment

Oklahoma Academic Standards Mathematics (Geometry)

G.2D.1.6: Use coordinate geometry and algebraic reasoning to represent and analyze line segments and polygons, including determining lengths, midpoints, and slopes of line segments.

Attachments

- Car Trouble (Sample Responses)—Meet Me in the Middle.docx
- Car Trouble (Sample Responses)—Meet Me in the Middle.pdf
- Car Trouble—Meet Me in the Middle - Spanish.docx
- Car Trouble—Meet Me in the Middle - Spanish.pdf
- Car Trouble—Meet Me in the Middle.docx
- Car Trouble—Meet Me in the Middle.pdf
- Exploring Population—Meet Me in the Middle - Spanish.docx
- Exploring Population—Meet Me in the Middle - Spanish.pdf
- Exploring Population—Meet Me in the Middle.docx
- Exploring Population—Meet Me in the Middle.pdf
- Lesson Slides—Meet in the Middle.pptx
- Story Problem Cards (Sample Responses)—Meet Me in the Middle.docx
- Story Problem Cards (Sample Responses)—Meet Me in the Middle.pdf
- Story Problem Cards—Meet Me in the Middle - Spanish.docx
- Story Problem Cards—Meet Me in the Middle - Spanish.pdf
- Story Problem Cards—Meet Me in the Middle.docx
- Story Problem Cards—Meet Me in the Middle.pdf

Materials

- Lesson Slides (attached)
- Exploring Population handout (attached; one per student; printed front/back)
- Story Problem Cards handout (attached; one per group; printed front only)
- Story Problem Cards (Sample Responses) document (attached; for teacher use)
- Car Trouble handout (attached; one per student; printed front only)
- Car Trouble (Sample Responses) document (attached; for teacher use)
- Student devices with Internet access
- Calculator (one per student)
- Colored pencils (2 colors per student)
- Ruler (one per student)
- Graph paper
- Scissors (optional)
- Document camera (optional)

15 minutes

Engage

Teacher's Note: Population Preparation

Prior to teaching the lesson, find out the number of students that attend your school, the population of your town, your state, your country, and the world. Update **slide 6** of the attached Lesson Slides with this data.

Consider asking someone in the main office about the school population. Population values for your town, etc. can be easily found through searching on the internet.

Use the attached **Lesson Slides** to guide the lesson. Review the essential question and objectives on **slides 3** and **4** with students. Transition to **slide 5** and ask students: "How much do we know about the population of our community?"

Use the Elbow Partner strategy and have students get out a piece of paper and work together to create an educated estimate for the following populations:

- our school
- our town
- our state
- our country
- our world

Provide time for students to estimate the populations.

Move to **slide 6** and provide students with the real number of your school's population and the actual population values of your town, state, country, and world.

Ask students to share their population amounts.

Teacher's Note: Guiding the Activity

For example, the population of the state of Oklahoma is roughly 4 million. Ask the class to hold up their hand if they estimate the Oklahoma state population as over 2 million. Ask them to keep their hand up if they estimated the state population as over 3 million, then 4 million, etc.

The population of the United States is roughly 330 million. Ask students how many had an estimate over that value. Then ask how many estimated less than that value.

Share with students that population data is gathered by the Census Bureau every ten years, and that this data is then used to create estimates for population in other years.

30 minutes

Explore

Move to **slide 7** and pass out the attached **Exploring Population** handout to each student as well as two different colored pencils and a ruler.

Instruct pairs of students to use the following websites (shortened URLs are provided on slide 7) to complete Part A of their handout:

- US Census Bureau national data per decade: Decennial Census of Population and Housing by Decades
- US Census Bureau regional and state data per decade: Historical Population Change Data (1910-2020)
- Town populations: It is recommended that students use Wikipedia.org and search for their town to locate the "Historical Population" listed.

When they are finished, direct students to put their devices away.

Show **slide 8** and direct students' attention to Part B and have them plot their data from Part A. Then have students connect their dots, using their ruler, to create a line graph of the data.

Display **slide 9** and have students use the data from Part A and Part B to estimate only the town's and state's populations for the years in the table for Part C. Guide students to not yet find the country's population.

40 minutes

Explain

Display **slide 10** and ask students to explain how they found their estimates for Part C. Ask students to raise their hand based on the approach they used to find their estimate.

- How many used their graph?
- Who used a formula?
- Who found those values using a different approach? What was your approach?

Continue until all approaches have been shared. Ask for a group to volunteer their reasoning and display their data using a document camera if necessary.

After discussing the different approaches, move to **slide 11**. Ask students to consider the discussed approaches and use one of the shared methods that was different from their original approach to find their country's population.

Display **slide 12** and direct students' attention to Part D of their handout. Use the Think-Pair-Share strategy and have students answer the question: "What do you notice about these points?"

Sample Student Responses: Part D

The points we plotted for Part C all fell on the line segments we drew on our graphs in Part B.

Transition to **slide 13** and direct students' attention to Part E. Again, using the Think-Pair-Share strategy, ask students: "What must be true about the year you are estimating the population for in order to use your method from Part C?"

Sample Student Response: Part E

The value we estimate must be halfway between two known values. Therefore, we couldn't use this method to estimate the population in 1983 since 1983 is not directly in the middle of two known years.

Next, have students finish Part E by writing a definition for midpoint (a word that hopefully came up during the earlier discussions) and writing a formula.

Teacher's Note: Guiding the Lesson

If students aren't using the word "midpoint," now is the time to give them that vocabulary.

Make sure all students agree that the point they estimate must be in the middle of two known values.

Transition to **slide 14**. Provide the definition of midpoint to students and have students write it at the bottom of their handout.

Ask students to work in their group and determine if the estimates they found in Part C fit the definition of midpoint. Make students aware that they must be able to explain how it fits or does not fit the definition and defend their reasoning. After providing time for students to think this through, have each group share their reasoning.

Ask the class what they think the tick marks tell us about the location of the midpoint. Emphasize to students the relationship between the tick marks and the location of the midpoint. If the two segments are congruent, then that middle point is exactly in the middle and is therefore the midpoint.

Teacher's Note: Guiding the Activity

Students might choose to simply compare the horizontal (x) distances between the estimates and endpoints or the vertical (y) distances. Make sure you ask them why they only compared distances in one direction, rather than two. They should actually determine the distances in both directions, but let them form conclusions on their own.

If students are struggling to see the connection between midpoint and the population activity, explain to them that they used the date between the two given dates (endpoints) to estimate the population for the middle year (midpoint). The **mid**dle of two points is the **mid**point.

Move to **slide 15**. Now that students have verified that their estimates are midpoints of line segments, tell them to continue to work with their partner to write a formula that allows them to find the coordinates (both x and y) of a midpoint using only the endpoints. Have them use the estimates they had already made in Part C to verify the formula they write.

Teacher's Note: Guiding the Activity

You might want to give them "generic" endpoint coordinates to use such as (x_1, y_1) and (x_2, y_2).

As groups finish creating and verifying their formulas, have each group share the formula they created by going to the board and writing their formula. After all the formulas are recorded, ask students to discuss which formulas seem the most reasonable and which seem to be most user-friendly. Have the class vote on one formula to use as the midpoint formula.

If their chosen formula is written differently than the standard formula, introduce students to the standard formula for midpoint by displaying **slide 16**. Have students compare the class-created formula and the standard formula for validity and ease of use. If the class formula is invalid, discuss the reasons why. If the class formula is valid, but in a different form, discuss the differences but stress that either method is valid (and allow them to use either on future assessments).

Teacher's Note: Connecting Prior Knowledge

Depending on the class, consider explaining that a median is just the midpoint of the range from a set of data.

30 minutes

Extend

Teacher's Note: Activity Preparation

Print the attached **Story Problem Cards** handout and cut out the six cards. Cut out one set of cards per group of students.

Put students into groups of 2–3 and move to **slide 17.** Ask students to think about how the midpoint formula can be used in real life. Provide some time for students to discuss in their groups for a few minutes. Ask for volunteers from each group to share their ideas and record their responses on the board or somewhere else where all students can see them.

Sample Student Responses

The midpoint formula could be used when planning a trip (a good place to stop for a break). It could be used in carpentry, construction, economics, etc.

Show **slide 18** and give each group of students a set of **Story Problem Cards** and a piece of graph paper and notebook paper. First, have students examine the six cards and only look for similarities and differences before solving. Ask for a few volunteers to share after students have looked at each card.

Teacher's Note: Purpose

Discussing the similarities and differences between these problems helps students to recognize when the midpoint formula is applicable and when it is not.

After providing time for students to examine the cards, tell students to begin solving the problem from the cards on a piece of notebook paper.

Every 10 minutes, or as needed, take a break from group work to call on each group to share an update of their progress. Ask each group to share a challenge they have discovered. As each group shares a challenge, ask the other groups if any of them have encountered the same challenge and, if so, how they solved it. Continue discussing the problems this way until all groups have shared a challenge. If a group says they have not yet encountered a challenge, have them share one of the solutions they have found instead. Encourage other groups to ask questions about the solutions presented. Continue in this manner until the groups have solved all six problems.

Sample Student Responses

Use the attached **Story Problem Cards (Sample Responses)** document for possible student responses.

10 minutes

Evaluate

Transition to **slide 19** and pass out the attached **Car Trouble** handout to each student to use with the Exit Ticket strategy. Be sure that they justify their answers using mathematical words and symbols. Collect the handout from students to assess understanding of the lesson content and inform future lessons.

Teacher's Note: Guiding the Assessment

Be sure that student explanations contain both a mathematical calculation for the midpoint between Dalhart, TX, and Tyrone, OK, as well as clear reasoning about the relative position of Texhoma to the midpoint.

See the attached **Car Trouble (Sample Response)** document for an exemplar student response.

Resources

- Bureau, U. S. C. (2021, October 8). *Historical Population Change Data (1910-2020)*. Census.gov. https://www.census.gov/data/tables/time-series/dec/popchange-data-text.html
- Bureau, U. S. C. (2021, November 24). *Decennial census of population and housing by decades*. Census.gov. https://www.census.gov/programs-surveys/decennial-census/decade.2020.html?utm_source=US%2BCensus%2BBureau
- K20 Center. (n.d.). Elbow Partners. Strategies. https://learn.k20center.ou.edu/strategy/116
- K20 Center. (n.d.). Think-Pair-Share. Strategies. https://learn.k20center.ou.edu/strategy/139
- K20 Center. (n.d.) Bell Ringers and Exit Tickets. Strategies. https://learn.k20center.ou.edu/strategy/125
- Wikimedia Foundation. (2022, June 17). *The Free Encyclopedia*. Wikipedia. Retrieved July 19, 2022, from https://www.wikipedia.org/