

We the People

Linear vs. Exponential Growth

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Grade Level	9th Grade	**Time Frame**	2-3 class period(s)
Subject	Mathematics	**Duration**	135 minutes
Course	Algebra 1		

Essential Question

How can population growth be modeled mathematically?

Summary

Students model population growth across continents to discover the difference between linear and exponential models. Students should already be familiar with both linear and exponential functions and equations.

Snapshot

Engage

Students read a brief news article about Boley, Oklahoma, and engage in a discussion about the relationship between the population of the town and how people live and work there.

Explore

Students gather data about the population of the United States. They then create both linear and exponential models for the data they collect and compare the two models for accuracy and precision.

Explain

Students determine whether linear or exponential models more accurately describe changes in population.

Extend

Students create exponential models for populations of five continents and make predictions based on these models.

Evaluate

Students read a brief article that introduces the concept of carrying capacity and create an exponential model of the world population to make a prediction about the number of years remaining before the Earth reaches its carrying capacity.

Standards

ACT College and Career Readiness Standards - Mathematics (6-12)

F702: Build functions for relations that are exponential

Oklahoma Academic Standards Mathematics (Algebra 1)

A1.F.2.1: Distinguish between linear and nonlinear (including exponential) functions. Understand that linear functions grow by equal intervals (arithmetic) and that exponential functions grow by equal factors over equal intervals (geometric).

Attachments

- Using Technology to create Lines of Best fit or Exponential Models - Spanish.docx
- Using Technology to create Lines of Best fit or Exponential Models - Spanish.pdf
- Using Technology to create Lines of Best fit or Exponential Models.docx
- Using Technology to create Lines of Best fit or Exponential Models.pdf
- We the People - Carrying Capacity - Spanish.docx
- We the People - Carrying Capacity - Spanish.pdf
- We the People - Carrying Capacity.docx
- We the People - Carrying Capacity.pdf
- We the People - Comparing Continents - Spanish.docx
- We the People - Comparing Continents - Spanish.pdf
- We the People - Comparing Continents.docx
- We the People - Comparing Continents.pdf
- We the People - Continental Populations - Spanish.docx
- We the People - Continental Populations - Spanish.pdf
- We the People - Continental Populations.docx
- We the People - Continental Populations.pdf
- We the People - Four Corners Signs - Spanish.docx
- We the People - Four Corners Signs - Spanish.pdf
- We the People - Four Corners Signs.docx
- We the People - Four Corners Signs.pdf
- We the People Datasheet - Spanish.docx
- We the People Datasheet - Spanish.pdf
- We the People Datasheet.docx
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Materials

- Internet access
- Microsoft Excel or graphing calculators
- Continent Populations handout (attached)
- Comparing Continents handout (attached)
- Carrying Capacity handout (attached)
- We the People Data Sheet (attached)
- Sticky notes
- Highlighters
- Pencils/pens
- Four Corners signs (attached)
- Sticky easel pad paper
- Markers (Mr. Sketch, Sharpie, etc.)

Engage

Direct students to the article about Boley, Oklahoma, or pass out printed copies of the article. Have students complete a Why-Lighting activity to answer the question "Why did the author of this article include information about the population of Boley? What point do you think they were trying to make?"

Teacher's Note

Students should recognize that the author included population information to illustrate how the town has declined. In other words, the population can help us understand the "health" of a town.

Have students complete a Think-Pair-Share activity over what they highlighted in the reading. Ask students what consequences the declining population of the town has on the remaining residents. Discuss this with them to illustrate the relationship between population and the everyday lives of citizens.

Teacher's Note

The purpose of this discussion is to have students form an understanding that it is important to be aware of population changes and what those changes mean to the real lives of people.

Explore

Distribute a copy of **We the People Datasheet** for every student. Have students work in groups of two or three to gather the data needed to complete the population column of the chart for Part A.

Teacher's Note

Make sure students note that the independent variable is written as years since 1900. This means that the row marked year "0" is 1900, and the year marked "10" is 1910 (and so on).

Teacher's Note

The data can be found here, but students may also refer to population data estimates from other sources. If the data estimates are not the same, it's a good opportunity to discuss why data estimates may differ and whether or not those differences are significant, given the large values of population.

Have students create a line of best fit for the data.

Teacher's Note

The directions for creating a line of best fit in Microsoft Excel (or on a graphing calculator) are attached to this lesson as "Using Technology to create Lines of Best fit or Exponential Models."

Students should now complete Part B on the document by using the line of best fit they created in Excel or on their graphing calculators. They can enter the equation for the line of best fit given on their chart in the appropriate space indicated; then, they should use the formula to enter the values predicted by this model into the spaces of the chart.

Before students complete Part C, discuss the concept of error with your students. Ask students to create (or recall) a formula to measure error. Discuss this formula until the class agrees on an accurate formula. Students should record this formula on their worksheets and then complete Part C.

Teacher's Note

A good measurement for error should: 1) consider the difference between the modeled value and the actual value and 2) compare this difference to the size of the actual value in a ratio. A typical error formula might be (Modeled Value - Actual Value) / (Actual Value). This model could also be multiplied by 100 so answers are expressed as percentages.

After students complete Part C, ask the class if they believe their linear model was accurate. Have them justify their reasoning orally.

Ask students to describe the physical meaning of the slope in the line of best fit. Lead them in a discussion until they reach the conclusion that the slope represents the change in the number of people living in the US each year.

Teacher's Note

Depending on the data used, the slope should be just over 2. This should be interpreted to mean that 2 million more people lived in the US each year.

Ask students to consider the population of the US in 1900 and 2010. Based on these populations, is it reasonable to expect that the increase in the number of people in the US was the same (approximately 2 million) each of these years?

Sample Student Responses

There were approximately 76 million people in the US in 1900. If the population increased by 2 million in that year, that would be one new person for every 38 people. By 2010, there were approximately 309 million people in the US. If the population increased by 2 million in 2010, then this means there was only one new person for every 155 people. This does not seem reasonable.

Ask students if they believe a linear model accurately represents population growth. Hopefully, they are convinced that it does not.

Ask students to think of types of functions that might better fit population data. Lead them in a discussion until they propose exponential functions as a possible alternative. Have students give reasons exponential functions might provide a better fit, based on their prior knowledge.

Teacher's Note

Good questions to ask during this discussion include: Why would that function work better? What would the coefficients/constants in that kind of function represent in this context? If we used that kind of function, how would the population change each year? Does that fit with our data?

Sample Student Responses

For exponential functions, we keep multiplying by the same number, instead of adding the same number. That means the change in population will get bigger as the population gets bigger.

Demonstrate how to create exponential curves of best fit on the TI-84 and in Excel. Directions on how to make exponential curves of best fit on TI-84 and on excel are attached to this lesson.

Have students complete parts D and E of the worksheet in groups of two or three.

Teacher's Note

If completing the work in Excel, the exponential model they create will be in the form P=C(e^(rt)). If creating the model on a TI-84 (or 83), the model will be in the form P=C(b^t). Alternative directions for each option are given in the remainder of this lesson. However, you should have all students complete the lesson using the same technology, so as to avoid confusing the students.

Explain

After all groups have completed their worksheets, have students discuss whether they believe the exponential model better fits their data in their groups. Then, have each group share the equation of the exponential model they created and the conclusions they reached. Record the equations of the models each group created on the board.

Teacher's Note

If using Excel, be sure that each model is of the form P=Ce^(rt), where P is population in millions and t is years since 1900. If using TI-83s or TI-84s, be sure that each model is of the for P=C(b^t)

After each group has shared their model, write the generic equation for an exponential model (P=C(e^(rt)) or P=C(b^t)) on the board. Ask students to identify the value of C in their model and then to explain the significance of C.

Teacher's Note

C should represent the population at time t=0. The worksheet has been structured so that t=0 represents the year 1900. So, C represents the population of the United States in 1900.

If using TI-83 or TI-84, skip to the next step. If using Excel, have each group calculate the value of e^r for their specific value of r. Students should label this value b for base.

Have each group report out their value of b. Ask the class what they believe these values of b represent. Lead the class in a discussion until they reach the conclusion that the value of b represents the percent change in population per year and is written as one plus the decimal equivalent of the percent change.

Ask students to once again consider the years 1900 and 2010 and determine if it is reasonable to believe that the percent growth rates are equivalent for these two years.

Teacher's Note

The growth rate should be approximately 1.25% per year, depending on the data used. In 1900, there were 76 million people. That means the model predicts that the population increased by approximately 76(0.0125) = 0.95 million people. This represents one new person for every 80 people. In 2010, there were approximately 309 million Americans. The model then predicts a growth in population of 209(0.0125) = 3.8625 million people, or one new person for every 80 people. This should seem like a more reasonable way to think about population growth.

Ask students if they believe exponential functions can accurately model population growth. Have them explain why they do or do not think the exponential models are accurate.

Extend

Hand each student two sticky notes. Write the names of the five major continents (Africa, Asia, Europe, North America and South America) in a horizontal line across a whiteboard. Direct students to place one sticky note above the name of the continent they believe has the largest population and the other sticky note below the name of the continent they believe has the lowest population.

Write the following prompt on the board: "An exponential model is written P=Ce^(rt). Both C and r represent unknown values. Since there are only two unknown values for this model, only two data points are needed to write an accurate exponential model for a given situation." Use this prompt to complete a Four Corners activity. The signs needed for the Four Corners activities are attached to this lesson.

Making Predictions

Having students make predictions about topics that can be described mathematically helps students to understand how we can have misconceptions and how mathematics can be used to address those misconceptions.

Tell students you are going to try to write models given only two data points and determine if the models created are reasonable. Group students into pairs or threes and give each group one section of the attached handout "We the People - Continental Populations." Each section contains data about a different continent, so be sure to distribute the sections evenly so that there are approximately the same number of groups working on each continent's data.

Teacher's Note

All students will also need a copy of "We the People - Comparing Continent" later in this lesson. These can be copied onto the back of the "We the People - Continental Populations" to save paper.

Have students work together to complete their section of the worksheet. Then, have all groups who are working on the same continent meet to share their results. Each content group should discuss their data and analysis until they agree on all solutions. This entails choosing a single equation to model the population of the continent.

Pass out the **We the People - Comparing Continents** handout if it was not copied onto the back of the We the People - Continental Populations handout.

Ask a group to share the model they decided upon and their reasoning. Ask each group whether they think the model they created is an accurate model for the population growth of their continent.

Teacher's Note

The models of each continent may be very different, but their reasoning should be similar. Students should value data that covers longer periods of time as well as more current data. That is, generally, the model created using data from 1960 and 2010 will be most valuable, since 2010 is very current data and since the data covers a range of 50 years.

Have all students copy the model given for each continent onto the We the People - Comparing Continents handout. Students can then work with their groups to find the current population of that continent. Have all groups record their answers using clickers or whiteboards as a formative assessment.

Continue calling on continent groups until each continent has been discussed.

Have students work in groups of two or three to answer questions 1-10 on the handout.

Once all students have completed their handouts, as for volunteers to share their answers. Allow students to check their work on questions 1-10, discussing the solutions when necessary.

Refer back to the results of the sticky notes predictions. Ask students to consider the predictions that the class made as they respond to question 11 on the handout. Give the students several minutes to respond to this question, then discuss their responses with them.

Possible Student Comments On Question 11

"We hear a lot more news about (Continent A) than (continent B), so I thought (continent A) was larger." and "on a map (Continent A) looks much bigger than (Continent B)" are two common students responses to question 11.

Evaluate

Distribute the **We the People - Carrying Capacity** handout. You may choose to have students complete this assignment individually or in pairs. They can either complete it in class or at home over several nights.

Resources

- Raymond, Ken. "Endangered Black History Series/Boley, Oklahoma". NewsOK. Copyright 2012, NewsOK and The Oklahoman. http://ndepth.newsok.com/black-history/boley
- "Population Estimates, Historical Data pre-1980 to 2010". (2012, December 20). United States Census Bureau. Copyright 2012, U.S. Department of Commerce. https://www.census.gov/popest/data/historical/
- K20 Center. (n.d.). Think-Pair-Share. Strategies. https://learn.k20center.ou.edu/strategy/139
- K20 Center. (n.d.). Why-Lighting. Strategies. https://learn.k20center.ou.edu/strategy/128
- Layton, Julia. "Has Earth reached its carrying capacity?". How Stuff Works Science. Copyright 2016, HowStuffWorks,a division of InfoSpace Holdings LLC. http://science.howstuffworks.com/environmental/green-science/earth-carrying-capacity1.htm