

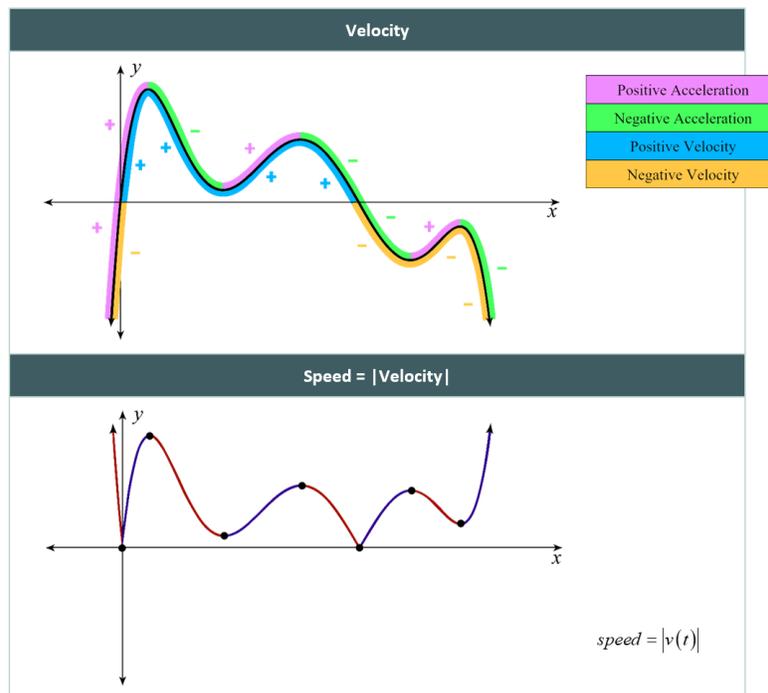
GUIDED NOTES (TEACHER GUIDE)

GUIDED NOTES (MODEL NOTES)

$s(t)$ = position function $s(t)$ = position

$v(t) = \frac{\Delta s}{\Delta t}$ as $\Delta t \rightarrow 0$ $s'(t) = v(t)$ = velocity

$a(t) = \frac{\Delta v}{\Delta t}$ as $\Delta t \rightarrow 0$ $s''(t) = v'(t) = a(t)$ = acceleration



Speed, on its own, measures how fast an object is traveling. Velocity represents both the speed and direction traveled.

After reviewing or introducing position, velocity, and acceleration, have students use 1 color to highlight or shade the positive acceleration rectangle to start the key. Then use that same color to draw **above** the curve everywhere that has a positive slope. Use that same color to draw a plus sign beside that interval. Repeat with a second color for the negative acceleration. Now use a third color to highlight or shade the positive velocity rectangle and draw **under** the curve where the velocity is positive. Repeat with a fourth color for negative velocity.

After introducing speed as the absolute value of velocity. Have students use 1 color to draw on or above the curve everywhere that has a positive slope. Use a second color to draw on or above the curve everywhere with a negative slope.

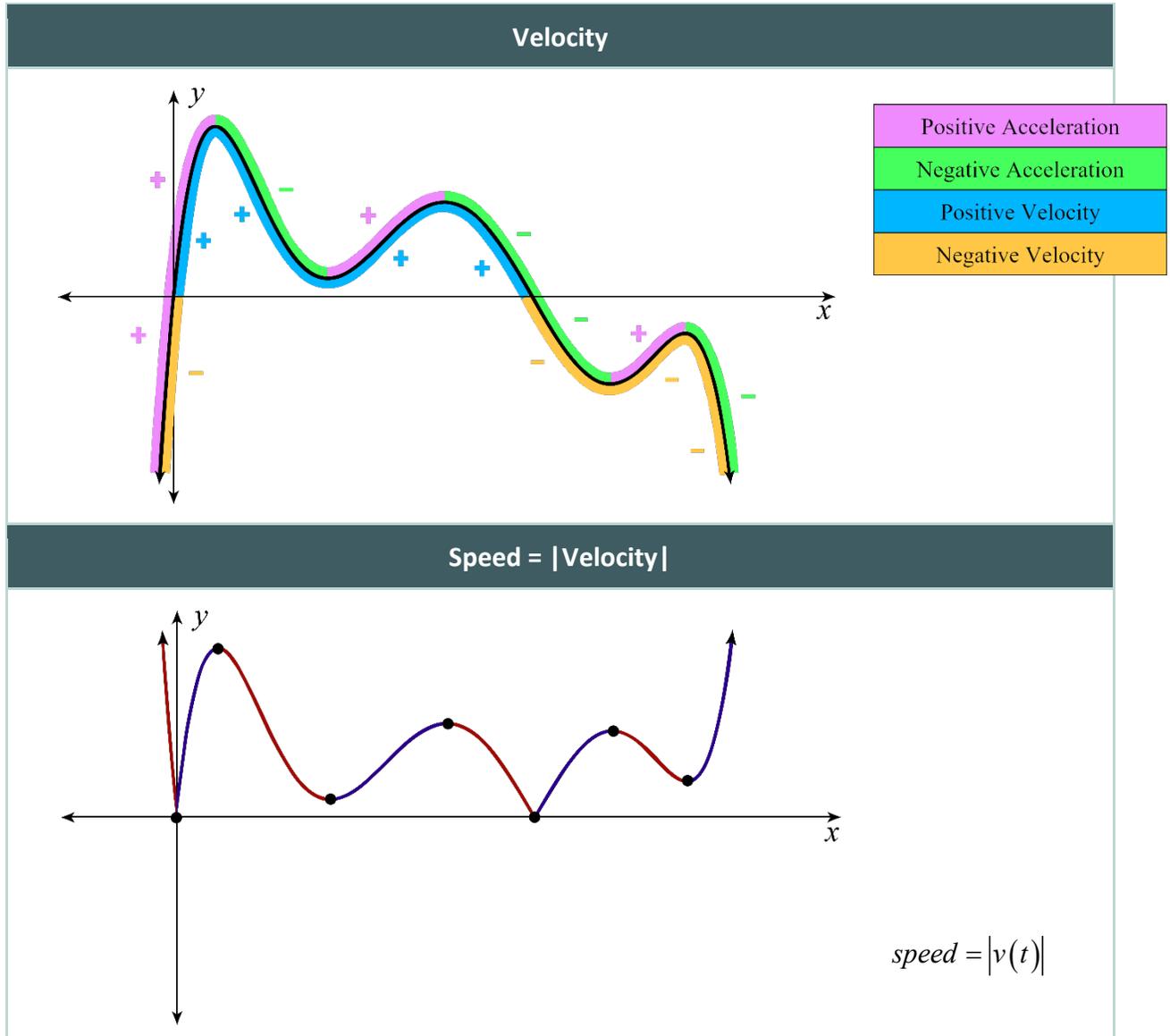
Have students observe the relationship between the two graphs. They should observe that the speed is increasing (the object is speeding up) when the slope of velocity (acceleration) has the same sign as the y-value of velocity. Similarly, when acceleration and velocity have different signs, the object is slowing down (speed is decreasing).

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Example Problems

The function $s(t) = 5 + \cos\left(\frac{\pi t}{2}\right)$ on the closed interval $[0, 3]$ models a particle's vertical motion along a line.

- 1) At what value(s) of t is $v(t) = 0$? Describe the particle's motion.

$$v(t) = s'(t) = -\sin\left(\frac{\pi}{2}t\right) \cdot \frac{\pi}{2}$$

$$0 = \sin\left(\frac{\pi}{2}t\right)$$

$$\frac{\pi}{2}t = 0, \pi \Rightarrow t = 0, 2$$

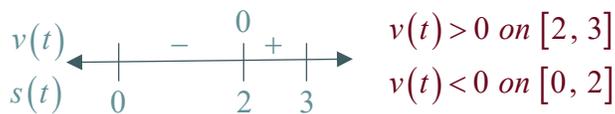
- 2) At what value(s) of t is $a(t) = 0$?

$$a(t) = v'(t) = -\frac{\pi}{2} \cos\left(\frac{\pi}{2}t\right) \cdot \frac{\pi}{2}$$

$$0 = \cos\left(\frac{\pi}{2}t\right)$$

$$\frac{\pi}{2}t = \frac{\pi}{2}, \frac{3\pi}{2} \Rightarrow t = 1, 3$$

- 3) On what interval(s) is $v(t) > 0$? $v(t) < 0$? Describe the particle's motion.



The particle is moving to the left on $[0, 2]$ and to the right on $[2, 3]$.

- 4) On what interval(s) is $a(t) > 0$? $a(t) < 0$?



- 5) Based on these results, when is the particle speeding up on the time interval? When is the particle slowing down? Justify your answers.

The particle is speeding up on $[0, 1]$ and $[2, 3]$ b/c that's when $v(t)$ and $a(t)$ are both negative and both positive, respectively.