Making Motion Matter

Connecting Position, Velocity, and Acceleration

Cacey Wells, Michell Eike, Ryan Rahhal
Published by *K20 Center*

Grade Level	12th Grade	**Time Frame**	95-110 minutes
Subject	Mathematics	**Duration**	2-3 class periods
Course	AP Calculus		

Essential Question

How are position, velocity, and acceleration related?

Summary

In this lesson, students will learn the relationship between position, velocity, speed, and acceleration to solve real-world problems involving motion along a line. Students are expected to know the chain rule and how to find higher-order derivatives for both power and trigonometric functions before beginning this lesson.

Snapshot

Engage

Students recall average rate of change through engaging with a real-world example.

Explore

Students run simulations, modeling vertical and horizontal motion, and see how they relate to their corresponding position and velocity graphs.

Explain

Students complete guided notes with the class and formalize their understanding of how position, velocity, speed, and acceleration are related.

Extend

Students apply what they have learned in order to sketch and match graphs of velocity and acceleration curves given position graphs.

Evaluate

Students demonstrate their understanding of the relationship between position, velocity, speed, and acceleration by answering a free response question.

Standards

AP Calculus AB and BC Course and Exam Description (AP Calculus AB & BC (2020))

CHA-3: Derivatives allow us to solve real-world problems involving rates of change.
CHA-3.B: Calculate rates of change in applied contexts.
CHA-3.B.1: The derivative can be used to solve rectilinear motion problems involving position, speed, velocity, and acceleration.

Attachments

- AP Calculus Free Response - Motion—Making Motion Matter - Spanish.docx
- AP Calculus Free Response - Motion—Making Motion Matter - Spanish.pdf
- AP Calculus Free Response - Motion—Making Motion Matter.docx
- AP Calculus Free Response - Motion—Making Motion Matter.pdf
- Guided Notes (Teacher Guide and Model Notes)—Making Motion Matter.pdf
- Guided Notes—Making Motion Matter - Spanish.docx
- Guided Notes—Making Motion Matter - Spanish.pdf
- Guided Notes—Making Motion Matter.docx
- Guided Notes—Making Motion Matter.pdf
- Sample Responses—Making Motion Matter.pptx

Materials

- Guided Notes handout (attached; one per student; printed front/back)
- Guided Notes (Teacher Guide and Model Notes) document (attached; for teacher use)
- AP Calculus Free Response – Motion handout (attached; one per student; printed front only)
- Sample Responses (attached)
- Coloring instruments (6 colors per student; highlighters, colored pencils/pens, etc.)
- Pencils
- Paper
- Student devices with internet access

5 minutes

Engage

Teacher's Note: Spanish Handouts

The learner handouts are available in English and Spanish to meet your students' needs. Keep in mind that the AP exam is only administered in English.

Teacher's Note: Desmos Classroom Activity Preparation

To use this Desmos Classroom activity, select the following link: "Making Motion Matter." Create an account or sign in under the "Activity Sessions" heading. After you log in, the green "Assign" dropdown button will be active. Click the arrow next to the word "Assign," then select "Single Session Code." After making some setting selections, select "Create Invitation Code" and give the session code to students. For more information about previewing and assigning a Desmos Classroom activity, go to https://k20center.ou.edu/externalapps/using-activities/.

For more detailed information about Desmos features and how-to tips, go to https://k20center.ou.edu/externalapps/desmos-home-page/.

To set up the activity's pacing for students, select "View Dashboard" (next to the session code). In the upper-left corner of your screen, select the icon above the word "Pacing." Desmos Classroom should then prompt you to select the first and last screens that you want students to see. When prompted to set a range, select screens 1 and 3. Select "Restrict to Screens 1–3" to confirm your selection. This allows students to access only screens 1–3 at this time. For more information about teacher pacing, go to https://k20center.ou.edu/externalapps/pacing-activities/.

Provide students with your session code. Then, have students go to student.ampify.com/join and enter the session code.

Teacher's Note: Sign-in Options

If students sign in with their Google or Desmos accounts, then their progress is saved, and they can resume the activity or view their work later. If students continue without signing in, they can complete the activity, but they must do so in one sitting. It is strongly recommended that students sign in; otherwise, they risk losing their work.

Direct students' attention to **screen 1** and then read the following prompt as the Bell Ringer:

An officer pulls you over and tells you that you were going 85 mph, and the posted speed limit is 80 mph. You think of a great idea—if you can quickly calculate your average speed, you might be able to convince the officer that your average speed was below the speed limit, and, therefore, he should let you off. You notice on your odometer that you have traveled 100 miles in the past 45 minutes. What was your average speed?

Have students calculate the average speed and type their answer and reasoning into screen 1.

Use this prompt to discuss average rates of change compared to instantaneous rate of change. Listen for misunderstandings to see if students need a quick refresh on instantaneous rates of change.

As time allows, ask for volunteers to share how they found their average speed and whether they think they could use this idea to get out of a ticket. (Students will also have the ability to see their peers' responses in the Desmos Classroom activity.)

Use **screen 2** to display the lesson's essential question and **screen 3** to identify the lesson's learning objectives. Review each of these with students to the extent you feel necessary.

15 minutes

Explore

Instruct students to find a partner or assign students partners. On the Dashboard, press the orange plus sign three times to allow students to progress to **screens 4–6**. Have students press the "Run" button and work with their partner to observe the relationship between the van's movement and the graphs of position and velocity. Then they are to read the graphs to answer questions about the direction the van is moving and the initial velocity.

Teacher's Note: Purpose

The purpose of screens 4–6 is for students to focus on motion along a horizontal line, to see how the position and velocity functions model that movement, and to reason through some possibly new ideas: initial velocity and horizontal direction of movement. As students progress, the complexity of the movement increases. Then, on screen 6, the movement of the van is mostly hidden, and students are expected to rely on the position and velocity curves to answer questions.

As students finish screen 6, bring the class together for a whole-class discussion. Ask for volunteers to share how they found the initial velocity and what they noticed about the graphs when the van was moving to the right. Use this time to correct any misconceptions.

On the Dashboard, press the orange plus sign four times to allow students to progress to **screens 7–10**. Have students press the "Run" button and work with their partner to observe the relationship between the hot air balloon's movement and the graphs of position and velocity. Then they are to read the graphs to answer questions about the direction the balloon is moving and the maximum height.

Teacher's Note: Purpose

The purpose of screens 7–9 is for students to focus on motion along a vertical line, to see how the position and velocity functions model that movement, and to reason through some possibly new ideas: maximum height and vertical direction of movement. As students progress, the complexity of the movement increases. Then, on screen 9, the movement of the hot air balloon is mostly hidden, and students are expected to rely on the position and velocity curves to answer questions.

As students finish screen 9, bring the class together for a whole-class discussion. Ask for volunteers to share how they found the maximum height and what they noticed about the graphs when the balloon was moving up. Use this time to correct any misconceptions.

30 minutes

Explain

Screen 10 indicates that students are to set aside their Desmos Classroom activity to complete the Guided Notes with the class. Give each student a copy of the attached **Guided Notes** handout. Introduce or review the definitions of *position*, *velocity*, and *acceleration*.

Give each student 6 coloring instruments. Students could share the 6 highlighters, markers, or colored pencils/pens. Have students use four of the colors to color-code the slope and *y*-values of the given velocity function: *positive acceleration* (slope), *negative acceleration, positive velocity* (*y*-values), and *negative velocity*.

Then have students use two colors on the graph of speed to indicate a *positive* slope (increasing speed) and *negative* slope (decreasing speed). Ask students to see if they could use the velocity graph to determine when the speed of an object is increasing or decreasing, without using the graph of the speed curve.

Help students see that when the signs of the *velocity* and *acceleration* are the same, the object speeds up, while the object slows down when velocity and acceleration have different signs.

Continue to complete the handout as a class.

Teacher's Note: Guiding the Lesson

Velocity is the change in position over the change in time: $v = (s_2 - s_1)/(t_2 - t_1)$. If students struggle to recall this, encourage them to think of the speed of a vehicle. We measure speed in miles per hour, so that is miles (distance or position) divided by hours (time). Students might also recall the formula distance = rate · time.

For more support and recommendations as you walk students through the Guided Notes, see the attached **Guided Notes (Teacher Guide and Model Notes)** document.

Once finished, have students add this to their math notebooks if that is a classroom norm.

30 minutes

Extend

On the Dashboard, click the orange "Stop" button; now students can complete the Desmos activity at their own pace.

Screen 11 previews the work of screens 12–25 where students will examine the relationships between a particle's position, velocity, and acceleration.

On **screens 12–17**, students are seeing motion that can be modeled by a linear position function. On **screens 18–25**, students are seeing motion that can be modeled by a quadratic position function.

Screen 12 starts with asking students to describe the motion of a green dot. Then screen 13 has students sketch the position-time curve to model the movement of the dot. Students receive feedback on screen 14 with the correct position-time graph. They are now to sketch the velocity-time curve and explain their thinking. Students receive feedback on screen 15 and are asked to sketch the graphs if the dot were moving twice as quickly and explain their thinking.

Using the Card Matching strategy, students are to match position-time graphs with their corresponding velocity-time graphs on screen 16, where they will see "Well done!" at the top of their screen if they match the cards correctly. After successfully completing the card match, students review screen 17 that transitions them into the next set of motion problems.

If students do not correctly match the cards on slide 16, they will see, "Go back to the previous screen and try again."

15 minutes

Evaluate

Once students have completed screen 25, direct their attention to **screen 26**. Give each student the **AP Calculus Free Response – Motion** handout and direct students to work independently so that you can individually assess what students have learned from this lesson. This question is intended to be solved without the use of a calculator.

Alternative Handout

This question is also displayed on screen 26 of the Desmos Classroom activity. Feel free to have students reference the question there and record their answers on their own sheet of paper.

Give students 10–15 minutes to answer the free response question. Then use the attached **Sample Response** slides to review the sample response and scoring guidelines with the class. Be sure to help students understand how to earn points on exam questions.

Teacher's Note: Source

This free response question is adapted from the 2010 AP Calculus AB (Form B) Question 6. It has been modified to assess what is addressed in this lesson. The scoring rubric has been modified accordingly.

Resources

- CollegeBoard AP. (2010). *AP Calculus AB 2010 Scoring Guidelines Form B*. Retrieved June 22, 2022, from https://secure-media.collegeboard.org/apc/ap10_calculus_ab_form_b_sgs.pdf
- K20 Center. (n.d.). Bell Ringer and Exit Tickets. Strategies. https://learn.k20center.ou.edu/strategy/125
- K20 Center. (n.d.). Card Matching. Strategies. https://learn.k20center.ou.edu/strategy/1837
- K20 Center. (n.d.). Desmos Classroom. Tech Tools. https://learn.k20center.ou.edu/tech-tool/1081
- Memed_Nurrohmad. (February 28, 2019). Car [Illustration]. Pixabay. https://pixabay.com/illustrations/car-transportation-vehicle-bus-4025379/