

Expanding Binomials, Expanding Your Mind (Part 1)

Connecting Pascal's Triangle to Binomial Expansion

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Grade Level	10th – 11th Grade	**Time Frame**	2-3 class period(s)
Subject	Mathematics	**Duration**	100 minutes
Course	Algebra 2		

Essential Question

What patterns emerge in Pascal's triangle, and how can they connect to binomial expression expansion?

Summary

In this lesson, students will have opportunities to explore patterns found in Pascal's triangle with the intent of using these patterns to expand binomial expressions. This lesson is Part 1 of 2. The goal of this lesson is to look at connections between binomial expansion and Pascal's triangle. Part 2 connects this lesson to the binomial theorem.

Snapshot

Engage: Students investigate the patterns that form Pascal's triangle.

Explore: Students work together to expand binomials using traditional methods and compare their results to what they see in Pascal's triangle.

Explain: Students work in groups to articulate the patterns they see and share with their classmates.

Extend: Students work in groups on more complicated binomial expansion using their newly constructed knowledge of Pascal's triangle.

Evaluate: Students self-assess their work and complete an Exit Ticket.

Standards

ACT College and Career Readiness Standards - Mathematics (6-12)

A505: Add, subtract, and multiply polynomials

Oklahoma Academic Standards Mathematics (Algebra 2)

A2.A.2.2: Add, subtract, multiply, divide, and simplify polynomial expressions.

Attachments

- Binomial-Expansion-Exploration-Expanding-Binomials-Expanding-Your-Mind - Spanish.docx
- Binomial-Expansion-Exploration-Expanding-Binomials-Expanding-Your-Mind - Spanish.pdf
- Binomial-Expansion-Exploration-Expanding-Binomials-Expanding-Your-Mind.docx
- Binomial-Expansion-Exploration-Expanding-Binomials-Expanding-Your-Mind.pdf
- Blank-Pascals-Triangle-Expanding-Binomials-Expanding-Your-Mind - Spanish.docx
- Blank-Pascals-Triangle-Expanding-Binomials-Expanding-Your-Mind - Spanish.pdf
- Blank-Pascals-Triangle-Expanding-Binomials-Expanding-Your-Mind.docx
- Blank-Pascals-Triangle-Expanding-Binomials-Expanding-Your-Mind.pdf
- Lesson-Slides-Expanding-Binomials-Expanding-Your-Mind.pptx
- Window-Notes-for-Math-Expanding-Binomials-Expanding-Your-Mind - Spanish.docx
- Window-Notes-for-Math-Expanding-Binomials-Expanding-Your-Mind - Spanish.pdf
- Window-Notes-for-Math-Expanding-Binomials-Expanding-Your-Mind.docx
- Window-Notes-for-Math-Expanding-Binomials-Expanding-Your-Mind.pdf

Materials

- Binomial Expansion Exploration handout (attached; one per student)
- Blank Pascal's Triangle handout (attached; one per student)
- Window Notes for Math handouts (attached; one per student)
- Pens or pencils
- Scratch paper

Engage

Classroom Setup Note

For this lesson, students will be working individually, in pairs, and in groups of four. Arrange desks or tables so that students are sitting in groups of four.

Teacher's Note: Concept Overview

In this lesson students will explore patterns in Pascal's triangle and connect these patterns to expanding binomials. The following videos provide an overview of these concepts.

https://www.khanacademy.org/math/precalculus/x9e81a4f98389efdf:polynomials/x9e81a4f98389efdf:binomial/v/pascal-triangle-binomial-theorem

https://www.youtube.com/watch?v=s19dWIHficY

Introduce the lesson using the attached Lesson Slides. **Slide 3** displays the lesson's Essential Question: *What patterns emerge in Pascal's triangle, and how can they connect to binomial expression expansion?* **Slide 4** identifies the lesson's learning objectives. Review each of these with your class to the extent you feel necessary.

Display **slide 5**. To begin, distribute the attached **Blank Pascal's Triangle** handout to each student.

With students, construct the first three or four rows of Pascal's triangle. DO NOT TELL STUDENTS THE PATTERN OR HOW TO FIND THE PATTERN. This is something they will discover as they continue to engage in the activity.

Ask students to work in pairs to see if they can determine the patterns needed to fill in the missing numbers for the remaining triangle.

Once students determine the pattern of Pascal's triangle, ask them to share with another pair of students.

As a whole class, invite students to share what they believe the pattern to be.

Clarify

Clarify misconceptions regarding the patterns formed in Pascal's triangle through probing questions. These questions allow students space to articulate the patterns they see emerging. For example, if students are struggling to see the big picture, ask them to only look at the first line and second line of the triangle and give descriptions of what they see. Once they do that, ask them what operation (addition, subtraction, multiplication, squaring, etc.) was used to get from the first line to the second line. Does it happen with every number? Do some numbers stay constant?

Explore

Setting Up The Exploration

Print the Binomial Expansion Exploration handout prior to beginning this activity.

Remember, the goal of this lesson is to help students make connections between expanding binomials and the patterns found in Pascal's triangle. For this exploration, students might get frustrated with expanding binomials, but there is a purpose to the pain! Your students are going to expand generic binomials in the form of (a+b)n using traditional methods in order to connect to the patterns within Pascal's triangle.

Distribute copies of the attached **Binomial Expansion Exploration** handout.

Tips And Tricks

In the handout, students might get bogged down with expanding something like (a+b)4. If this is the case, ask probing questions to lead students to rewriting (a+b)4 as (a+b)2 * (a+b)2. If they are not understanding, go back and ask them how to use the FOIL method to expand a basic binomial. From there, ask them what mathematical process they will have to use if the degree is higher than two. Can you just FOIL or do you have to do something else after?

Go to **slide 6.** Ask students to work in pairs or in their groups of four to expand the binomials on part one (1) of the handout.

Be sure to move around the room to help students clarify their work, asking probing questions to help them arrive at the correct binomial expansions.

When students finish part 1, have them move on to part 2 of the handout. Part 2 asks students to make connections between coefficients in their expanded binomials and what they see in Pascal's triangle.

Explain

Go to slide 7. In order to clarify misconceptions about the topic, ask students to complete part 3 of the Binomial Expansion Exploration handout.

For part 3, students will follow the instructions in the handout to do the following:

- Find a partner in a different group.
- Compare and contrast their answers to parts 1 and 2 of the handout.
- Return to their original groups to share new insights they might have learned.

Extend

Teacher's Note: Setup

Print copies of the Window Notes for Math handout to use with the Extend activity. Before students embark on Part four, distribute copies of the **Window Notes for Math.**

Display **slide 8.** Ask students to work in their small groups to complete part 4 of the handout. This activity is intended to move students past the generic binomial form to more complex problems.

As students work through part 4, ask them to fill in the Window Notes handout squares.

Evaluate

Display **slide 9**. In small groups, ask students to share the questions they have from the Binomial Expansion and Window Notes handouts.

As questions come up, ask students to help their peers who have questions come to an understanding of the problems at hand.

Ask students to take out a blank sheet of paper. As students leave for the day, ask them to reflect on their learning using the What? So What? Now What? strategy.

- What? Literally, what did you do?
- So what? Why does it matter?
- Now what? What will you take with you? How do you think this might connect to other areas of mathematics?

Teacher's Note: Understanding the Gaps

While the students are answering the What? So What? Now What? as an exit ticket, pay attention to their answers. Students need to master the expansion of binomials in order to move on to arithmetic sequences and summation. They should be writing about patterns and how they can identify patterns to find bigger problems like summation and difference formulas. They may not use the exact terminology, but they should be close. If they are not writing those types of responses, take time to review before moving on.

Resources

- K20 Center. (n.d.). What? So what? Now what?. Strategies. https://learn.k20center.ou.edu/strategy/b30762a7557ba0b391f207f4c6002113
- K20 Center. (n.d.). Window notes. Strategies. https://learn.k20center.ou.edu/strategy/fc74060730ea745c8c4f356aa2015ac0
- Khan Academy. (n.d.) Pascal's triangle and binomial expansion. [Video]. https://www.khanacademy.org/math/precalculus/x9e81a4f98389efdf:polynomials/x9e81a4f98389efdf:binomial/v/pascals-triangle-binomial-theorem
- The Organic Chemistry Tutor. (2016, December 4). Binomial Theorem Expansion, Pascal's Triangle, Finding Terms & Coefficients, Combinations, Algebra 2. YouTube. https://www.youtube.com/watch?v=s19dWIHficY