Things to Do with a Strip of Paper: Hexaflexagons

Translations, Reflections, & Trapezoids

Grade Level	6th – 10th Grade	**Time Frame**	2-3 class period(s)
Subject	Mathematics	**Duration**	100 minutes
Course	Geometry, Middle School Mathematics		

Essential Question

How do symmetries found in hexagons translate to other shapes and transformations?

Summary

In this lesson, students will create and explore mathematics found in hexaflexagons. This includes learning more about translations, rotations, and reflections of polygons, showing how these types of transformations preserve congruency, and learning to calculate area of trapezoids and composite figures.

Snapshot

Engage

Students create a tri-hexaflexagon.

Explore

Students explore various facets of their tri-hexaflexagon.

Explain

Students explain the types of symmetry found in their tri-hexaflexagon and shapes that form the tri-hexaflexagon.

Extend

Students create and explore a hexahexaflexagon.

Evaluate

Students reflect on their learning.

Standards

Oklahoma Academic Standards for Mathematics (Grade 6)

6.GM.4.1: Predict, describe, and apply translations (slides), reflections (flips), and rotations (turns) to a two-dimensional figure.
6.GM.4.2: Recognize that translations, reflections, and rotations preserve congruency and use them to show that two figures are congruent.

Oklahoma Academic Standards for Mathematics (Grade 6)

7.GM.2.1: Develop and use the formula to determine the area of a trapezoid to solve problems.

Attachments

- Exploring Your Tri-Hexaflexagon - Spanish.docx
- Exploring Your Tri-Hexaflexagon - Spanish.pdf
- Exploring Your Tri-Hexaflexagon.docx
- Exploring Your Tri-Hexaflexagon.pdf
- Hexa-hexaflexagon Template .pdf
- Hexaflexagon Template.pdf
- What So What Now What - Spanish.docx
- What So What Now What - Spanish.pdf
- What So What Now What.docx
- What So What Now What.pdf

Materials

- Plain copy paper
- Markers
- Pens
- Hexaflexagon and Hexahexaflexagon templates (attached)
- Exploring Your Tri-Hexaflexagon handout (attached)
- \What So What Now What handout (attached)
- Tape or glue
- Scissors
- A flat work surface

Engage

Students make a hexaflexagon.

Various Levels Of Structure

Hexaflexagons can be constructed with or without the aid of a template. Depending on where your students are in their knowledge of equilateral triangles, using the provided template can be a helpful aid in the construction process. Feel free to consult videos from Vi Hart for inspiration and guidance.

As students are learning to construct a hexaflexagon, the way in which you guide them can be more or less hands-on. The two methods described below are extremes, and you can use your own discretion to provide as little or as much guidance and scaffolding as your students need.

More Hands-OFF Method: Distribute a sheet of paper to each student. Allow students to cut out an even strip of paper to their satisfaction. Keep in mind that some will work and some won't. (Again, this is an exploratory method that is more hands-off.) Have students determine how they can divide the strip of paper into 10 equilateral triangles (triangles that have three equal sides and three equal angles). Ask students to fold them up to create their hexaflexagons. This method might require more time, comfort with students feeling frustrated, and a flexible (pun intended) level of guidance when students are struggling. Help students as they need it, but first try asking questions to help guide students. For example, if a student's hexaflexagon isn't working, ask, "Have you considered folding in a different way?"

More Hands-ON Method: Allow students access to the attached hexaflexagon template. Play the videos provided on Vi Hart's website to help show students how to construct their hexaflexagons. Show struggling students how to fold to get a good crease. Show students how to fold the hexaflexagon so that it flexes. There is obviously more showing here, but students are still doing. You might want to have students try one on their own after your initial guidance.

The goal with either method is for students to do the constructing. The way you go about working with your students, though, is entirely up to you!

Explore

Arrange students in groups of three and assign each a role: Recorder, Sharer, or Questioner.

- Recorders will document the group's answers.
- Sharers will explain the group's answers to the whole class.
- Questioners will read questions to the group and pose new questions.

Tell students that they will be exploring the hexaflexagons that they constructed to uncover the hidden side.

Working in their groups of three, ask students to assume their assigned roles and work through the questions in the Exploring your Tri-Hexaflexagon handout. All students are expected to participate and describe what shapes they see within the hexaflexagon. Remind students that the questioners read the questions, recorders record the answers, and sharers share out to the whole class.

Variation Of Answers

The questions in the handout/slides are intended to be open-ended. For example, when asked how many triangles they see, some students might count only one face, some might count two, and others all three. Some might see larger triangles composed of multiple smaller triangles. Allowing students to explain their varying answers can lead to great discussion.

Explain

Ask each group's sharer come to the front of the room to share their answers with the whole class.

Follow-Up Questions

As students share their work with the group, jot down questions that are worth investigating further. This might be a discrepancy in the number of triangles, the way students described symmetry, etc. You'll use these questions to help students explain their reasoning. For example, you might note: "Marty saw there were six triangles, but Jerome found there to be twelve. How is this possible?" Or: "Julio noticed that the sides of the hexaflexagon sometimes don't bend to reveal the inside hexagon. Why do you think that is?" The questions can be as open or closed as you like, keeping in mind that the purpose is to allow students to explain the phenomena.

After each group presents, open the floor for discussion as a whole class.

Keep The Conversation Alive

If discussion as a whole class sounds more like crickets than students, change up the direction of your discussion. Consider using a Think-Pair-Share instructional strategy to provide safe space for students to discuss with a partner or small group.

Extend

This section is similar to the Explore, but this time there is less analysis of the hexaflexagon and more construction.

Ask students to construct a hexa-hexaflexagon. To do this, provide students with the hexa-hexaflexagon handout, along with scissors and tape or glue. Give them the option to color their construction.

Try It Without A Template

If students' confidence levels are high after making their original hexaflexagons, give them the option to try this one without a template as well. Will it be difficult? You bet! Problem-solving, though, is not always an easy process. Use your best discretion.

Evaluate

Please, Please Don't Collect Students' Hexaflexagons

Flexagons are meant to be played with! Sharing in the mathemagical construction that is the hexaflexagon is half the fun! Allow students to take these with them, encourage them to make more, and follow up in a day or two to ask if they've expanded anyone's minds with the awesome power of the hexaflexagation! Instead of collecting students' work, ask them simply to reflect on their time exploring this super-fun mathematical concept using a What? So What? Now What? strategy.

Pass out copies of the What? So What? Now What? handout and have students respond to the following prompts:

- What? What did you do in today's activity?
- So what? What did you learn in today's activity and how does it connect to mathematics?
- Now what? What will you take with you from this lesson? (This obviously could be the hexaflexagon itself, but you're looking for a more conceptual takeaway.)

Resources

- Hart, V. (n.d.). Hexaflexagons. Retrieved from http://vihart.com/hexaflexagons/
- Gathering4Gardener (2017). Hexahexaflexagon Blank Template. Retrieved from http://www.puzzles.com/hexaflexagon/img/hexahexaflexagon_blank_template.pdf
- K20 Center. (n.d.). Think-Pair-Share. Strategies. https://learn.k20center.ou.edu/strategy/d9908066f654727934df7bf4f5064b49
- K20 Center. (n.d.). What? So What? Now What?. Strategies. https://learn.k20center.ou.edu/strategy/b30762a7557ba0b391f207f4c6002113