# Thinking Outside of the Box

## Surface Area and Volume of Rectangular Prisms

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| **Grade Level** | 9th – 10th Grade | **Time Frame** | 85-95 minutes |
| **Subject** | Mathematics | **Duration** | 2-3 class periods |
| **Course** | Geometry | | |

## Essential Question

What is the relationship between surface area and volume? How are surface area and volume used in the real world?

## Summary

In this lesson, students explore concepts of perimeter, area, surface area, and volume while engaging in hands-on construction of origami boxes. Students collaborate to discover the formulas for volume and surface area. Then students listen to a civil engineer and learn how he uses volume and surface area in his job before solving a real-world problem.

## Snapshot

**Engage**

Students make predictions about how the area of a square piece of paper once it is folded might compare to the surface area and volume of a box.

**Explore**

Students explore the relationships between perimeter, area, surface area, and volume of origami boxes.

**Explain**

Students critically think about and discuss the relationships between perimeter, area, surface area, and volume.

**Extend**

Students learn how a civil engineer uses volume and surface area then solve a real-world problem.

**Evaluate**

Students reflect on their learning by using the Muddiest Point strategy.

## Standards

*ACT College and Career Readiness Standards - Mathematics (6-12)*

**G601:** Use relationships involving area, perimeter, and volume of geometric figures to compute another measure (e.g., surface area for a cube of a given volume and simple geometric probability)

*Oklahoma Academic Standards Mathematics (Geometry)*

**G.3D.1.1:** Represent, use, and apply mathematical models and other tools (e.g., nets, measuring devices, formulas) to solve problems involving surface area and volume of three-dimensional figures (prisms, cylinders, pyramids, cones, spheres, composites of these figures).

## Attachments

- Constructing Origami Boxes—Thinking Outside of the Box - Spanish.docx
- Constructing Origami Boxes—Thinking Outside of the Box - Spanish.pdf
- Constructing Origami Boxes—Thinking Outside of the Box.docx
- Constructing Origami Boxes—Thinking Outside of the Box.pdf
- Engineering the Bridge Cap—Thinking Outside of the Box - Spanish.docx
- Engineering the Bridge Cap—Thinking Outside of the Box - Spanish.pdf
- Engineering the Bridge Cap—Thinking Outside of the Box.docx
- Engineering the Bridge Cap—Thinking Outside of the Box.pdf
- Exploring Origami Boxes—Thinking Outside of the Box - Spanish.docx
- Exploring Origami Boxes—Thinking Outside of the Box - Spanish.pdf
- Exploring Origami Boxes—Thinking Outside of the Box.docx
- Exploring Origami Boxes—Thinking Outside of the Box.pdf
- Lesson Slides-Thinking Outside the Box.pptx

## Materials

- Lesson Slides (attached)
- Constructing Origami Boxes handout (attached; one per student; print front/back)
- Exploring Origami Boxes handout (attached; one per student; print front/back)
- Copy paper (2 per student, each cut into 8.5” square)
- 1 cm cubes (approx. 200 per pair) or 1 inch cubes (approx. 20 per pair)
- Rulers (one per student)
- Dice (1 for teacher use)
- Pencils
- Paper
- Individual dry erase boards (optional; one per student)
- Dry erase markers (optional; one per student)
- Chart paper (optional; one per class)

15 minutes

# Engage

**Teacher's Note: Lesson Preparation – Square Paper**

Before beginning the lesson, cut 8.5"x11" copy paper into 8.5" squares. Each student needs 2 square papers for this lesson. Consider making a few extra for each class in case a student needs a replacement.

Introduce the lesson using the attached **Lesson Slides**. **Slide 3** displays the lesson's essential questions. **Slide 4** identifies the lesson's learning objectives. Review each of these with your class to the extent you feel necessary.

Display **slide 5** and distribute one sheet of square paper to each student. Guide students to use this paper for measuring and recording their thinking; alternatively, have students use an individual dry erase board for recording.

Share the Think-Pair-Share strategy with your students and ask the prompts: *How big is your paper? In what ways can you measure it?*

**Teacher's Note: Perimeter and Area**

The idea of phrasing the questions above in a vague manner is to lead students toward measuring the perimeter and area of their papers. Some students might just measure across or measure diagonally. These are great ways to measure. If students do not come up with measuring the perimeter on their own, ask them how big around their square is. If students do not come up with the idea to measure the area, engage them in a conversation about area, what it is, and how to measure it.

Ask students to think independently about finding the area and perimeter of their square paper. Have them record their thinking and answers on that square sheet of paper.

Display **slide 6** and have students pair up to share their work and their answers with their partner. Transition to **slide 7** and have a few student pairs share with the class.

Next, show **slide 8** and pose the following question: *Given the area and perimeter, how much can this paper hold?*

Encourage students to be creative to answer this question. They can fold, cut, tear, etc. the paper to try to create a box and estimate its volume. Give students approximately 5 minutes to complete this task.

Have students record their thinking on a piece of notebook paper or on an individual dry erase board.

If time allows, ask a few volunteers to share their thoughts with the whole class.

**Teacher's Note: Guiding the Activity**

Try not to explicitly tell students to create a box. This is time for them to think creatively about the posed question. This is not yet the time to give students the rulers or cubes.

25 minutes

# Explore

Show **slide 9** and distribute the **Constructing Origami Boxes** handout. Then give each student a new square sheet of paper. Direct each student to create their own origami box following the directions on the handout.

### Additional Resources

To meet the needs of your students in constructing an origami box, consider sharing the following video or handout from The Japan Society website: https://www.japansociety.org.uk/resource?resource=76.

Once students are almost done constructing their boxes, distribute rulers and cubes to students to use as manipulatives to help measure the volume of their box in the next step of the activity.

Display **slide 10** and give each student a copy of the attached **Exploring Origami Boxes** handout. Direct students' attention to the *Making Predictions* section of the handout. Have them answer the following questions and record their answers on their handouts:

1. How much volume do you think your box can hold? How can you measure this without a formula?
2. How much surface area do you think your box has?
3. How could you measure this?

Direct pairs to continue following the directions on their handout to complete the activity. Here students are using rulers and cubes to help them measure the volume and surface area of their boxes. Then they are to use their answers to questions 3-7 to complete the *Developing Formulas* table.

### Teacher's Note: Guiding the Activity

The goal of the *Developing Formulas* table is for students to notice a relationship between perimeter, area, volume, and surface area and to use those relationships to observe patterns and create general formulas for volume and surface area of a rectangular prism.

As pairs are working through the table, circulate through the room and ask different pairs of students some of the following questions, as this is what they should be discussing with their partner to complete the table.

1. Share how you measured the perimeter of the base of your box. What did you measure? How did you measure it? Can you come up with an equation for the perimeter of the base of your box?
2. Share how you measured the area of the base of your box. What did you measure? How did you measure it? Can you come up with an equation that represents the area of a square?
3. Share how you measured the volume of the origami box you created. What did you measure? How did you measure it? Can you come up with an equation that represents the volume of a rectangular prism?
4. Share how you measured the surface area of your box. How many sides did you measure? Would your answer change if the box had a top/lid? How so? Can you come up with an equation for the surface area of a rectangular prism?

20 minutes

# Explain

Show **slide 11** and ask students to consider the following question: "What relationships do you see between the answers you put in your table?"

Introduce the Critical Thinking Cube strategy to the class and explain that you want them to think about this question as they complete the next activity.

Preview the activity by explaining that you will roll a dice and share out the number. Students will then have about 2 minutes to talk with their partner about the corresponding prompt. Call on pairs to share with the whole class. For clarity, explain to the class that if a 3 is rolled, they will discuss prompt #3 (which they will see on the following slide) with their partner for 2 minutes and then share with the whole class.

### Digital Dice

Instead of using a physical dice, consider using the CPM Probability Generators to roll a digital dice.

Transition to **slide 12** and share the following prompts:

1. **Describe:** Describe how you found the volume of your box.
2. **Apply:** Write a formula to find the volume of any box.
3. **Compare:** Compare volume to surface area.
4. **Analyze:** What are the dimensions of volume? Of surface area?
5. **Synthesize:** Write a formula to find the surface area of any box (with a lid).
6. **Argue:** "Is there a relationship between area, perimeter, volume, and surface area." Justify your answer.

During the whole class share out, record the common big idea of the class for each prompt on the board or a designated spot on the chart paper. The goal is for students to understand the concepts of volume and surface area and to be able to find the volume and surface area of any rectangular prism and later any prism.

After discussing prompt #2, show **slide 13** and help students see the connection between the two formulas for volume. Explain to students that the volume of any prism is the area of the base times the height.

After discussion prompt #5, show **slide 17** and help students understand this likely new formula for surface area. Take time to discuss the definition of the lateral faces. Consider having students label the lateral faces of their origami boxes.

### Teacher's Note: Guiding the Lesson

Use the hidden **slides 14-16** and **18-21** for guidance on how to help students make connections if they need any additional scaffolding.

Use slides 14-16 as guidance for helping students understand the difference between area and volume, how they are related, and the units used for volume.

Use slides 18-21 as guidance for helping students understand the relationship between area, perimeter, and surface area.

Help ensure students understand the big ideas from this lesson: the volume of a prism is the area of the base times the height, and surface area is the sum of the areas of each surface.

20 minutes

# Extend

### Teacher's Note

Decide whether you want the Engineering the Bridge Cap handout to be guided practice or independent practice. The sample responses to the handout are on hidden slides, so if you would like the class to check their work as they go, unhide slides 25-28.

If you choose independent practice, plan to have students submit their handout and use it as a formative assessment.

Show **slide 22** and ask the class, “What are some careers you can think of that would use surface area, volume or both in their job?” Use the 30-second timer on the slide to give all students a chance to think about all the careers that may use surface area and volume in their job. Once the timer expires, have students quickly share their ideas with the whole class.

Show **slide 23** and introduce the “Civil Engineer and Rectangular Prisms” video, which features an interview with Bobby Williams, a civil engineer talking about his career and how he uses surface area and volume in his job.

**Embedded video**

https://youtube.com/watch?v=qrQ2Mrg6Vg0

Move to **slide 24** and pass out the attached **Engineering the Bridge Cap** handout to your students. This task represents a real-world problem posed by the civil engineer in the video interview. Depending on your preference or your students’ needs, you may consider having your students work with a partner or independently.

After students finish, transition to **slide 29** and pose the question: *If the emblem mold has a diameter of 5.2 feet, do we need a redesign?* Be sure to have students justify their conclusions.

### Optional Slides

Unhide and display **slides 25-28** so students can check their work. Ask for volunteers to explain their work.

**Teacher's Note: Additional Scaffolding**

If students struggle with the problem, consider the following:

*Help Starting:*

- Draw a picture.
- Label what you know. What information was given? What formulas should I use?
- Identify and write what you want to know. What is the question asking you to find?

*Help with Part A: Break It Down Into 2 Steps*:

- What is the weight of the original bridge pier cap?
- What is the maximum allowed weight without a redesign? How can we rewrite that first sentence algebraically?

*Help with Part B:*

- This question asks for the shape/size of the maximum allowed bridge pier cap. If we know the maximum allowed weight, how would we find the longest allowed dimensions?
- How does volume relate to the side length of the face?

5 minutes

# Evaluate

Go to **slide 30**. Have students reflect on the lesson and their overall understanding of the content using the Muddiest Point strategy. Have students answer the following questions:

- Crystal Clear: What do you think is the easiest part of finding volume and surface area?
- Muddiest Point: What do you think is the most confusing part of finding volume and surface area?

You can collect responses in a variety of ways depending on your class. Sticky notes, pieces of paper, or digital posts are a few examples.

## Resources


- K20 Center. (n.d.). Critical Thinking Cube. Strategies. https://learn.k20center.ou.edu/strategy/1583
- K20 Center. (n.d.). CPM Probability Generators. Tech Tools. https://learn.k20center.ou.edu/tech-tool/2317
- K20 Center. (n.d.). Muddiest Point. Strategies. https://learn.k20center.ou.edu/strategy/109
- K20 Center. (n.d.). Think-Pair-Share. Strategies. https://learn.k20center.ou.edu/strategy/139