Manipulating Mathematics: Building a Conceptual Foundation

Time Frame 180-240 session(s)

Essential Question(s)

- How can using manipulatives make mathematics more accessible to students?

Summary

This interactive and collaborative professional development session examines the importance of supporting and developing students' growth and mathematical mindset through the use of manipulatives. Participants will experience a model lesson exploring how mathematical thinking and conceptual understanding is fostered through the use of math manipulatives when intentionally used as authentic, student-centered tools for learning mathematical ideas. Participants will reflect on the learning, then demonstrate their understanding of how manipulatives support conceptual mathematical thinking and make it more accessible to all students when paired with non-routine mathematical tasks.

Learning Goals

- Participants will experience mathematical concepts by using manipulatives to solve non-routine tasks.
- Participants will explain how these experiences can make mathematical ideas more accessible to all students and develop students' conceptual foundations about mathematics.
- Participants will create a lesson or a non-routine task using manipulatives to make the concept more accessible and foster students' conceptual understanding.

Attachments

- 3-2-1.docx
- Box of Boxes (Activity).docx
- Box of Boxes Recording sheet and Little Boxes.pdf
- Estimating Fractional Parts (Activity).docx
- Fraction Cover Up (Activity).docx
- Fraction Strips Kit (Activity).docx
- Fraction Uncover Version 1 (Activity).docx
- Fraction Uncover Version 2 (Activity).docx
- Growing Shapes (Activity).docx
- Manipulating Mathematics PD.pptx
- Math Manipulative Centers Note Sheet.docx
- Mathematical Statements for Magnetic Statements .docx
- SCORE Reflection.docx
- Target Number (Activity).docx

Materials

- "Mathematical Statements" for the Magnetic Statements
- "Math Manipulative Centers" note sheet
- Activity and Manipulative Handouts found in the ATTACHMENTS
- Manipulatives and Activity handouts for "Probability Centers"
- Manipulatives and Activity handouts for the LEARN lesson, "Pythagor-eatin' Theorem"
- Notecards (four for each participant)
- "SCORE Reflection" note sheet

Engage

Presenter's Note: Session Prep

Have handouts and materials available on the tables for participants. All participants should receive a copy of the "Math Manipulative Centers" note sheet. Tables should be organized into small groups. Before the presentation, gather and organize all the mathematics activities and materials and clearly display the "Mathematical Statements" for the magnetic statements activity (found on slide three of the attached presentation) in different areas of the room.

Begin with **slide two**, displaying the professional development title. Welcome participants and briefly introduce yourself and the session. Objectives and goals will be addressed after the opening activity.

Transition to **slide three**, introducing the **"Mathematical Statements"** for the Magnetic Statements strategy. Participants will read the statements on **slide four** and identify the one that they are either most attracted to or most repelled by, then move to stand next to that statement. (Note: in a small session, participants may remain seated and select the statement they feel most attracted to or repelled by. These can be shared out individually to the whole group or they may turn and talk with an Elbow Partner.) Once participants understand the objective, display the statements and allow them time to find a statement to stand by. After groups have formed around the statements, instruct participants to share within their group why they chose that particular statement. One person will summarize what was discussed to the whole group.

Possible Responses:

"I was attracted to this statement (#2) because we learn from mistakes. They can be a powerful teaching tool," or "I was most repelled by the first statement because I don't believe that everyone can learn to the highest levels. Some people need more basic teaching and repetition of skills." (**Note:** if no one has selected the first statement, it is important to acknowledge that many educators believe the statement is false. Depending upon your participants, you might ask, "Considering statements two through seven, if we shifted our focus from a teacher-centered approach to more student-centered, would it open the door to make mathematics more accessible to all students and possibly allow all students to reach higher levels of mathematics?")

Ask participants to return to their seats. Then briefly share the essential question and learning objectives for the session, found on **slides five and six**. Participants should keep these in mind as they explore each activity using manipulatives. Transition to **slide seven**, reading the quote. Explain that we will begin exploring different mathematical activities that use manipulatives, but not all activities can foster deep thinking about mathematical concepts. It is important to be intentional with the tasks selected. Tasks and activities must open and provoke deep thinking and questioning about mathematical ideas, concepts, and relationships.

Explore

Presenter's Note: Considerations For Math Centers/rotations

Consider the following when setting up math centers/rotations: (1) How much time is needed for each activity or how much time is remaining in the session? For example, 10 minutes per activity or 15 minutes per activity. (2) How many participants are in the session? Fewer numbers may require no rotations, but the whole group engages together; larger numbers may require duplicates of each activity. (3) The number of materials you are able to access. Do you have enough dice to make multiple stations of the Target Number activity, for example, or do you need to create additional activities? (4) How will participants know when to transition between each activity?

Provide three to four activity and manipulative centers for the participant groups to rotate between. Each small group will explore each center for about 10 minutes and then use 5 minutes to reflect and answer the two prompts on the "Math Manipulative Centers" note sheet. Groups rotate after 15 minutes (time must be monitored and notify groups when the first 10 minutes have passed, and reflections should begin).

Transition to **slide eight**, introducing each of the mathematical activities and manipulatives that each group will be exploring during the rotations. Also, provide instruction regarding the **"Math Manipulative Centers"** note sheet. At each center, use this note sheet to record the following (shown on **slide nine** and on the top of the note sheet):

1. How does each manipulative develop and foster students' conceptual understanding and thinking about mathematical ideas?
2. How does each activity open mathematics, making it accessible to all students?

After the instructions have been given, have participants explore each activity in timed rotations. *(Note: see attached **"Activity and Manipulative Handouts"** for all optional rotations, resources, and instructions.)*

Explain

After participants complete the assigned centers, display **slide 10**, introducing the 3-2-1 strategy. Invite participants to individually reflect on the manipulatives and activities they explored, recording their thoughts on the designated **"3-2-1"** handout. After a few minutes of personal reflection, invite participants to share (either in small groups or as a whole) how manipulatives, in general, can make mathematics more accessible and foster students' conceptual mathematical thinking and mindset.

Connections

The main goal is to see how manipulatives are beneficial to students' access to and learning of mathematics. It may be helpful to remind participants that not all activities are created equal. Manipulatives should not be used solely by the teacher to display mathematical concepts or relationships. Manipulatives impact student learning when they are in the hands of students. They should be available for use as students need them to construct foundational and conceptual knowledge, thus making the abstract more concrete. Do not hide them in a closet. Rather, co-construct norms with students for using manipulatives.

Extend

After participants have reflected and shared the benefits of using manipulatives, transition to **slide 11**. Participants will now select a topic they plan to teach in the next two to three weeks and begin to develop a lesson outline/plan or a non-routine task that incorporates manipulatives. The lesson/task should utilize manipulatives to make mathematical ideas and relationships more accessible to all and fosters students' conceptual foundation. This is protected time to apply new knowledge into their practice.

Collaborative Groups For Planning/brainstorming

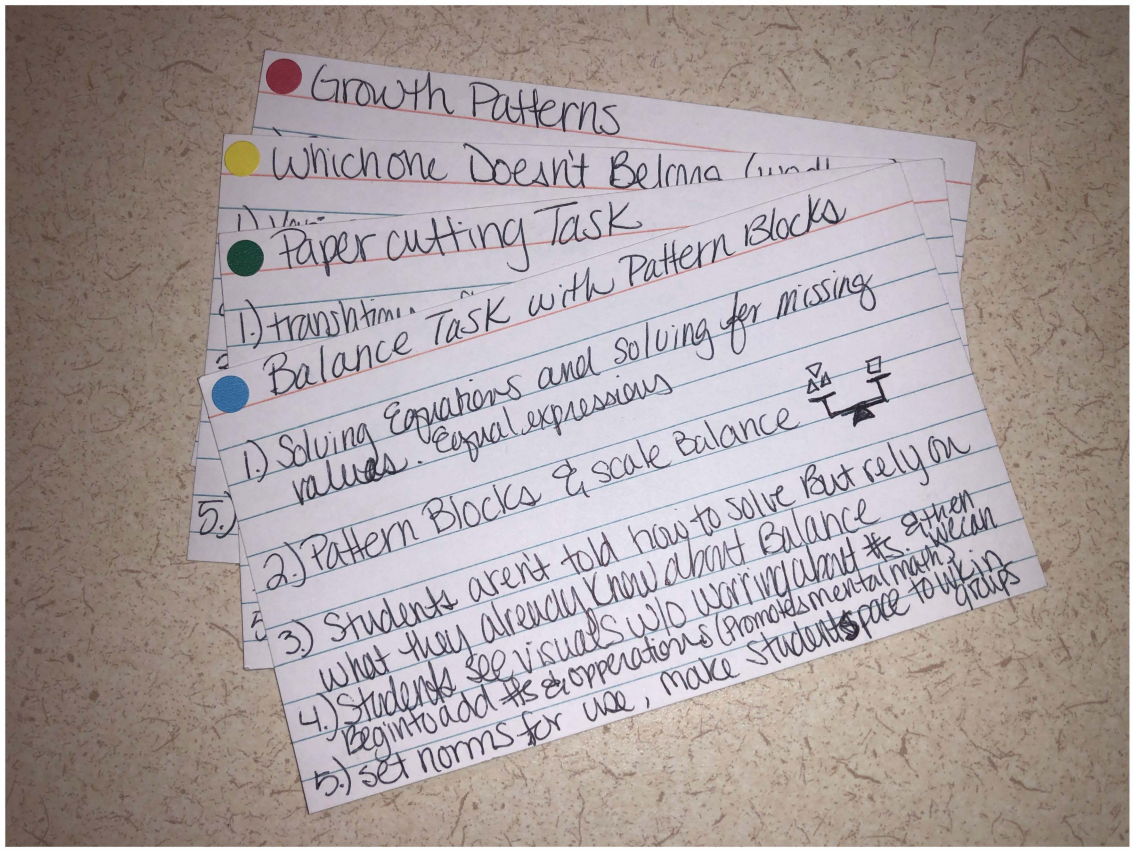
Organize participants into small groups based on grade level or topic. Participants can work together to construct a lesson/non-routine task that accomplishes the stated goals using manipulatives and brainstorm the questions on the next slide.

While groups work, display **slide 12**. Participants will summarize their lesson/task in preparation to share with other participants. Encourage participants to think deeply about numbers three and four as they will expand and connect to those ideas during the Evaluate portion of the session.

If participants need guidance, ask them some of the questions below:

- Which standards and mathematical ideas will be addressed by the lesson/task?
- How much time can you allot? *Be realistic, if the lesson/task takes a whole week and addresses one standard then it won't be worth it. However, if it addresses multiple standards and takes a week, ask yourself, "How deep are students learning the mathematical ideas?" It may be worthwhile if mathematical ideas are being explored deeper.*
- *How will manipulatives open mathematical ideas to all students, and how does the lesson/task develop and support their mathematical conceptual foundation?*
- How might you structure and prepare your class for this lesson/task? *Should the seating/student groups be rearranged? Do students need to co-create norms for using manipulatives?*

After participants have had at least 15 to 20 minutes of protected planning time, draw their attention to the notecards on the tables and change to **slide 13**. Participants will summarize their lesson/task using the details highlighted on slide 13 in preparation to share with other participants. *(Note: each participant will record the exact same thing on four notecards. These notecards will be traded with others during the following activity.)*



Example of completed notecards from four different participants.

Transition to **slide 14**. Have participants mix and mingle around the room, locating three other participants with whom they will briefly describe their lesson/task and then exchange one copy of their notecards. From this activity, participants will gain three additional lesson/task ideas that use manipulatives to make mathematics more accessible to students and develop students' conceptual understanding.

Evaluate

Finally, change to **slide 15**, instruct participants to revisit the **"Mathematical Statements"** introduced at the beginning of the session. Participants will now move to the statement they believe will be most supported by or associated with their lesson/task (or a lesson/task they received from another participant). Have participants stand by the chosen statement. *(Note: in a smaller session, participants may remain seated and select the statement they feel most strongly towards.)*

Instruct participants to share their reasoning for choosing this statement within the statement groups. Ask one person from each group to summarize what was discussed to the whole group, connecting back to why these lessons and tasks support the selected statement. *(Note: if participants struggle to connect to statement, ask them to specifically look at numbers three and four from the notecards. If they continue to struggle, ask them to consider how they might modify the lesson/task so at least one statement is better supported.)*

Presenter's Note For Hiden Slide 16

TREK evaluations will be used at the end of any GEAR UP Professional Development Session. Your Professional Development Coordinator at the K20 Center will provide you with a TREK number once you have notified them of the scheduled PD. The provided TREK number will be added to Slide 16, replacing the five number signs (#####).

Follow-up Activities

Presenter's Note: Score Reflections

Anywhere from a week to a month after the session, schedule and host an informal reflection with small groups of participants who attended the formal professional development session. This might be one session with all participants or multiple sessions with different participants each time. These follow-up sessions can be held during a PLC or teacher planning period since they are to be short and informal. Use slide 18 and the attached "SCORE Reflection Note Sheet" for this follow-up session.

Display **slide 17** as participants filter into the meeting. Once you and the participants are ready to begin the meeting, change to **slide 18**. Instruct participants to use the **"SCORE Reflection Note Sheet"** to record reflective notes about their own experience implementing any lessons and tasks that used the manipulatives explored from the professional development session.

After participants have reflected and recorded their thoughts, facilitate a small group share-out/discussion providing each attendee time to highlight the manipulatives they used and their students' overall experience. The questions on slide 18 mirror the "SCORE Reflection Note Sheet" and should be used to guide the discussion.

Encourage attendees to use another manipulative and continue to follow up with each participant, cultivating a safe environment for accountability and growth.

Research Rationale

Boaler (2016) suggests that one thing teachers must do in order to support and develop a mathematical mindset within students is to show them how mathematics is a visual and beautiful subject. Naturally, the human brain wants to think visually about mathematics. Therefore, instruction, opportunities to learn and explain mathematics visually must be presented to and explored by students. Complex mathematical ideas are abstract, but all mathematics originates from the concrete. Teachers must allow students time to construct meaning with the visual and concrete world of mathematics and manipulatives are one way for students to explore mathematics. We often move too quickly from the concrete to the abstract. Children need time to play, manipulate, and represent their thinking and understanding in order to fully process and engage in meaningful mathematics. Students should have time to create and discover mathematical patterns and relationships which connect to a larger mathematical idea or concept. Many researchers (Boaler, 2016; Boaler, 2014; Smith & Stein, 2018; Wheatley,1991) discuss how rich mathematical tasks, when properly supported by teacher facilitation and a carefully cultivated classroom environment which permits learning to be constructed in a natural, authentic process that is shared by students and teachers equally (Dewey, 1956, Freire; 1970, 2005; Boaler, 2016; Davis, 1997), can empower students who typically struggle in the traditional mathematics classroom. Furthermore, these tasks can change students' perspectives and mindsets about mathematics. Pairing manipulatives with rich mathematical tasks encourage engagement and make mathematics more accessible to all students.

Resources

- Boaler, J. (2014). Research suggests that timed tests cause math anxiety. Teaching Children Mathematics, 20(8), 469-474.
- Boaler, J. (2015). Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching. John Wiley & Sons.
- Davis, B. (1997). Listening for differences: An evolving conception of mathematics teaching. Journal for Research in Mathematics Education, 28(3), 355-376.
- Dewey, J. (1956). The child and the curriculum and the school and society (Combined ed.).
- Freire, P. (1970, 2005). Pedagogy of the oppressed 30th-anniversary edition. Translated by Myra Bergman Ramos. New York: Continuum.
- K20 Center. (n.d.). 3-2-1. Strategies. Retrieved from https://learn.k20center.ou.edu/strategy/d9908066f654727934df7bf4f5059a7b
- K20 Center. (n.d.). Elbow partners. Strategies. Retrieved from https://learn.k20center.ou.edu/strategy/ccc07ea2d6099763c2dbc9d05b00c4b4
- K20 Center. (n.d.). Magnetic statements. Strategies. Retrieved from https://learn.k20center.ou.edu/strategy/d9908066f654727934df7bf4f50761bf
- K20 Center. (n.d.). Pythagor-eatin' theorem. Constructing Pythagorean Theorem. Retrieved from https://learn.k20center.ou.edu/lesson/bb01792b8b7ae172f01b1c728a02b5ba
- Smith, M. S., and Stein, M. K. (2018). 5 practices for orchestrating productive mathematics discussions. Reston, VA: National Council of Teachers of Mathematics.
- Stein, M. K., Smith, M. S., Henningsen, M. A., & Edward, A. Silver. 2000. Implementing Standards-Based Mathematics Instruction: A Casebook for Professional Development.
- Wheatley, G. H. (1991). Constructivist perspectives on science and mathematics learning. Science Education, 75(1), 9-21.
- Youcubed. (n.d). Positive classroom norms poster. Ideas & impact. Growth mindset. Standford Graduate School of Education. Retrieved from https://bhi61nm2cr3mkdgk1dtaov18-wpengine.netdna-ssl.com/wp-content/uploads/2017/03/Norms-Poster-2015.pdf OR https://www.youcubed.org/resource/growth-mindset/